



Braille notation

mathematical

physical chemical

2nd edition

Publication recommended by the Ministry of National Education for use in preparing Braille versions of textbooks, exam papers, and supporting materials

Dedicated to the memory of the editor of the guide *Braille notation for mathematics, physics, and chemistry, Krakow 2002*

Jana Świerczka

and in memory of the pioneers
of Braille mathematical notation in Poland

***Andrzej Adamczyk
and Andrzej Galbarski***

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Edited by Jan Świerczek

Mathematical and physical notation

Krystyna Bodys
Ewa Fojucik
Maria Gackowska
Bogumiła Golańska
Wiesław Grabowiec
Tomasz Kępa
Anita Kluska
Jan Krempa
Bogusław Kwiatek
Marian Magner
Dobromiła Ofiarska-Kwiatkowska
Małgorzata Placha
Zbigniew Różga
Tadeusz Rynkowski
Elżbieta Szwedowska
Maryla Woźniak

Chemical notation

Antoni Audycki
Krystyna Broniarz
Anna Kolosowska
Jolanta Kondyjowska-Ogórek
Lucyna Mełgieś
Jadwiga Namysłak
Ilona Piotrowska
Stanisława Tarnawska
Elżbieta Zakroczyńska

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Decisions of the notation team *Workshop, January 7–8, 2011, Łódź*

Edited by Krystyna Kauba

Mathematical and physical notation

Tomasz Flaga – proofreading
Ciesielska – proofreading
records Marzena Sławińska – work coordinator
Katarzyna Budzanowska
Beata Cieślińska-Bodzioch
Katarzyna Chorodeńska
Ewa Dziewiątkowska
Doroła Jakubiuk
Krystyna Kauba
Anita Kluska
Jolanta Kozyra
Małgorzata Łydka
Iwona Pawłowska
Anna Ostrowska
Krystyna Waligóra
Maryla Woźniak
Małgorzata Zawadzka-Ostrowska

Chemical notation

Jolanta Kondyjowska-Ogórek Magdalena and Leszek Ogórek – compilation of
Alicja Bagińska
Krystyna Broniarz
Iwona Chmielarska
Marianna Gajowiec
Anna Kolosowska
Elżbieta Zakroczyńska

Polish Association of the Blind in Warsaw – consultations

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Mathematical Notation
Mathematical Notation for the Blind, the "Szansa" Foundation for the
Disabled

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Anna Tomaszewska

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Special Education Teachers at Educational Centers for Blind Children
on Braille notation,
Łódź, January 7–8, 2011

The Director of the Center in Laski

Piotr Grocholski

for organizing the National Conferences for Special Teachers at Centers for
the Blind *on Braille notation in mathematics, physics, and chemistry*, Laski,
June 12–13, 2008, and March 7–8, 2011

Authors

Louis Braille Special School and Educational Center
No. 1 for Visually Impaired and Blind Children and
Youth

ul. Zygmunta Krasińskiego 10
85-008 Bydgoszcz
www.braille.bydgoszcz.pl

Special Educational Center for Blind and
Visually Impaired Youth ul. Hajducka 22
41 - 500 Chorzów
www.sosw.slask.pl

Special Educational Center for Visually
Impaired and Blind Children named after
Zofia Książek-Bregułowa
ul. Wybickiego 1
41-303 Dąbrowa Górnicza
www.osw.dabrowa.pl

Włodzimierz Dolański Special Educational
Center for Blind and Visually Impaired
Children

ul. Tyniecka 6
30-319 Kraków
www.blind.krakow.pl

Educational Center for Blind Children
named after Róża Czacka in

Laski ul. Brzozowa 75
05-080 Izabelin
www.laski.edu.pl

Special Educational Center for Visually
Impaired Children and Youth
named after Prof. Zofia

Sękowska 20-092 Lublin
ul. Hirszfelda 6
www.sosw.type.pl

Special Educational Center No. 6 for Visually
Impaired and Blind Children
named after Major Hieronim

Baranowski ul. Dziewanny 24
91-866 Łódź
www.blind.edu.pl

Special Educational Center for Blind Children

Pl. Przemysława 9
62-005 Owińska
www.niewidomi.edu.pl

Lower Silesian Special Educational Center No. 13 for the Blind
and Visually Impaired
named after Maria

Grzegorzewska ul.
Kamiennogórska 16
54-034 Wrocław
www.oswdn.pl

A bit of history

The search for a means of communication between blind and sighted people has a very long history dating back to ancient times. In his collection *Disputationum Tusculanarum*, Cicero mentions the names of several enlightened blind people – Democritus, Appius Claudius, Aufidius, Diodot – who were widely admired and respected by their contemporaries. They certainly used the help of readers and scribes to pass on their knowledge. We can only assume that they attempted to become independent of third parties by searching for a means of communication that did not involve sight.

The first documented attempts to develop a writing system accessible to the blind date back to the 14th century. At that time, Zain-Din Al Amidi, a blind professor at the University of Iraq, developed a unique type of raised writing, which he used to mark the volumes of his extensive book collection and to take short notes.

Blind scholars also worked at European universities, and their activities certainly encouraged the search for a means of recording and transmitting information. An additional stimulus was the invention of printing at the end of the 15th century, which made books widely available to sighted people.

Francesco Lucas of Zaragoza (1517) proposed various versions of the idea of engraving letters of the alphabet on wooden tablets. In his opinion, blind people could learn the shape of letters by touch. Erasmus of Rotterdam (1528) presented a different concept. Although he also proposed engraving letters, he suggested doing so on a hard material, such as ivory or metal. According to him, blind people should run a stylus along the contour of a concave letter until they memorize the muscle movement of their hand. Once they have mastered this skill, they will be able to write by hand on paper.

It is worth noting here a characteristic feature of any fully-fledged writing system. The information should also be readable by the writer, which was lacking in many ideas for writing systems for the blind. The technique of writing also plays a significant role. Engraving or embossing the shape of letters is possible, but quite difficult, slow, and complicated.

An interesting initiative was proposed by the Jesuit Lan Terzi in 1670, who suggested a writing system combining lines intersecting at right angles and dots punched into paper arranged in individual fields. In this writing system, blind people could not only write, but also read information themselves. Lan's introduction of dots and frames to handwriting was an innovative solution that later played an important role.



Lana Terzi writing system

Mikołaj Saunderson, a mathematician and physicist who became blind in childhood and was a professor at Cambridge University, developed tables on which he presented information using pinheads after learning about Lana's system. According to Diderot, Sounderson was able to solve complex mathematical problems on his tables, check his work, and correct any mistakes he noticed.

Many blind people mastered techniques for recording information. However, most of them used cipher signs. Only a few mastered the Latin alphabet and used it. All of them belonged to the privileged class. The education of blind people was not organized in any way.

Significant changes in this field took place in the 18th century. New trends and theories concerning education that emerged at that time led to the search for methods of educating the blind. The pioneer was the Frenchman Valentin Haüy, a man with a strong social conscience. After successful attempts to teach the beggar François Leseur, he founded the first school for the blind in history in 1784. Valentin Haüy's experiment found many imitators. More schools were established to educate

the blind. Centers were established in England, in many European capitals, and also across the Atlantic. In Poland, too, a department for the blind was established in 1842 at the Institute for the Deaf and Dumb, which had existed since 1817.

All schools faced the same problem. They were desperately searching for a writing system accessible to the blind. However, the attempts made were based on the use of the writing system used by the sighted, which created insurmountable difficulties for the blind.

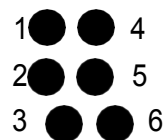
During this period, artillery captain Charles Barbier developed a cipher script for military purposes that could be read by touch even in the dark. Like Lan Terzi in the previous century, Barbier introduced a raised dot to encode the characters. Various codes were created for different recipients. The basis of Barbier's writing system is a vertical rectangle composed of two rows of six dots. The letters and sounds of the French language are encoded phonetically in this system. After developing a suitable ruler for pricking characters on paper, Barbier presented his invention in 1821 at the Institute for the Blind in Paris, offering it for use by the school.

The new writing system, introduced to the Institute on a trial basis, was enthusiastically received by the students, but met with more skepticism from the administration and teaching staff. The students could easily write themselves, do their schoolwork, take notes, and read them quickly. However, the phonetic method of writing made it impossible to use spelling, and there was no punctuation or numbers. A significant disadvantage of Barbier's writing system, despite the large number of raised dots used, was the small number of character-symbol combinations.

An interesting opinion on Barbier's proposal was expressed by a special commission of the French Academy of Sciences (represented by Lacépède and Ampère), which, after reviewing the writing system, stated in a special report: "Ordinary writing is an art that appeals to the eye, while the one invented by Charles Barbier is an art that appeals to the touch."

Some of the more talented and ambitious pupils at the institute sought ways to improve the writing system on their own. Among them was the young Louis Braille, who was admitted to the institute as a ten-year-old boy in 1819. After familiarizing himself with Barbier's writing, Braille intuitively sensed that this dot system, which differed from conventional letters, had a great future as a proper writing system for the blind. In 1824, he loyally presented his plan to reform the system to Captain Barbier, but the latter responded negatively to the proposal. Then, fifteen-year-old Louis set about introducing radical changes.

In the first stage, he reduced Barbier's sign to six dots, arranging them vertically in two rows of three, from which he created a six-dot system offering 64 combinations of the number and mutual arrangement of dots.



He arranged the characters obtained in this way into six series of ten and allocated the first four series to the Latin alphabet in order and letters with French diacritical marks. He allocated the fifth and sixth series to punctuation, numbers, etc.

The logical development of this system is that the first ten letters of the alphabet from *a* to *j* are represented by combinations of dots 1, 2, 4, 5. The next series of letters are created by successively adding first the third dot, then the third and sixth dots, and finally the sixth dot. The shape of a Braille letter can be felt immediately by a blind person with their fingertip, without counting individual dots.

After several years, thanks to his ingenuity and abstract thinking skills, Louis Braille, who was then a tutor and music teacher at the Institute, expanded his system to adapt it to mathematical notation, music notation, different languages, special symbols, etc.

The year 1837, when the second edition of Braille's instructions describing the principles of dot writing was published, can be considered the actual date of the creation of Braille. The system presented in it, with its simple structure, easy, transparent, and above all open, adaptable to foreign languages, based exclusively on six dots, was already well established.

Braille in Poland

- 1864–1865 Braille was known at the Institute for the Deaf and Blind in Warsaw. Due to reluctance and a lack of understanding of the issue, teaching Braille was abandoned after a few years. Klein's writing system was used instead.
- 1878 Braille was introduced at the Institute for the Blind in Lviv, adapted to the Polish language by Aleksander Schönfeld, a blind Polish emigrant in Paris.
- 1890 Braille was adapted to the Polish language and reintroduced at the Institute in Warsaw.
- 1925 The first national convention of special school teachers. The Polish Braille alphabet was established on the basis of the so-called "Bydgoszcz system" in use at the time.
- May 24, 1934. Decree of the Ministry of Religious Denominations and Public Education approving the Braille writing system for the Polish language developed by Elżbieta Róża Czacka and Teresa Landy according to Monnier's numbering system, as well as the system of Polish Braille abbreviations.
- November 4, 1981: Order of the Minister of Education and Upbringing on the introduction of a modified Polish alphabet in the Braille system and Braille spelling abbreviations in special schools for the blind and visually impaired.
- 1994 Andrzej Galbarski, *Handbook for transcribing books using the Braille system*, first edition, Polish Association of the Blind, Recording and Publishing House, Warsaw 1994.
- 2003 *Guide to the rules for using partially modified Braille notation in humanities publications. A guide for teachers, Braille instructors, publishers of Braille publications, and Braille users*. ZG PZN, Warsaw 2003.

Braille in mathematical notation

- 1929–1937 Work of the international commission on the development of a uniform system of Braille mathematical notation.
- 1941 Colonel Stafford *International Braille Code for Mathematics and Chemistry* England. The proposed coding system required further work.
- 1955 Helmut Epheser *International Braille Mathematical Notation for the Blind* Germany, Hannover-Marburg 1955. A compact and consistent notation system was developed. It was introduced in many countries.
- 1967 Dr. Helmut Epheser *International Braille Mathematical Notation*, translated and edited by Maria and Andrzej Adamczyk, Main Board of the Polish Association of the Blind, Warsaw 1967.
- 1986, 1992 Helmut Epheser *International Braille Mathematical Notation for the Blind* Germany, Hannover-Marburg, 1986 – Braille version, 1992 – black print version. Modified notation from 1955. New symbols were introduced and the system for writing multi-level expressions was changed.
- 1994 Andrzej Galbarski *Handbook for Transcribing Books into Braille* First edition, Polish Association of the Blind, Recording and Publishing House, Warsaw 1994.
- 2002 Collective work edited by Jan Świerczek *Braille notation for mathematics, physics, and chemistry – A guide for teachers* Krakow 2002.
- 2004 Marek Kalbarczyk, Dr. Jan Omieciński *You too can be a mathematician, Mathematical notation for the blind*, GUIDE The Union for Assistance to the Disabled "Szansa" Foundation.

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'S BRAILLE SYMBOLS

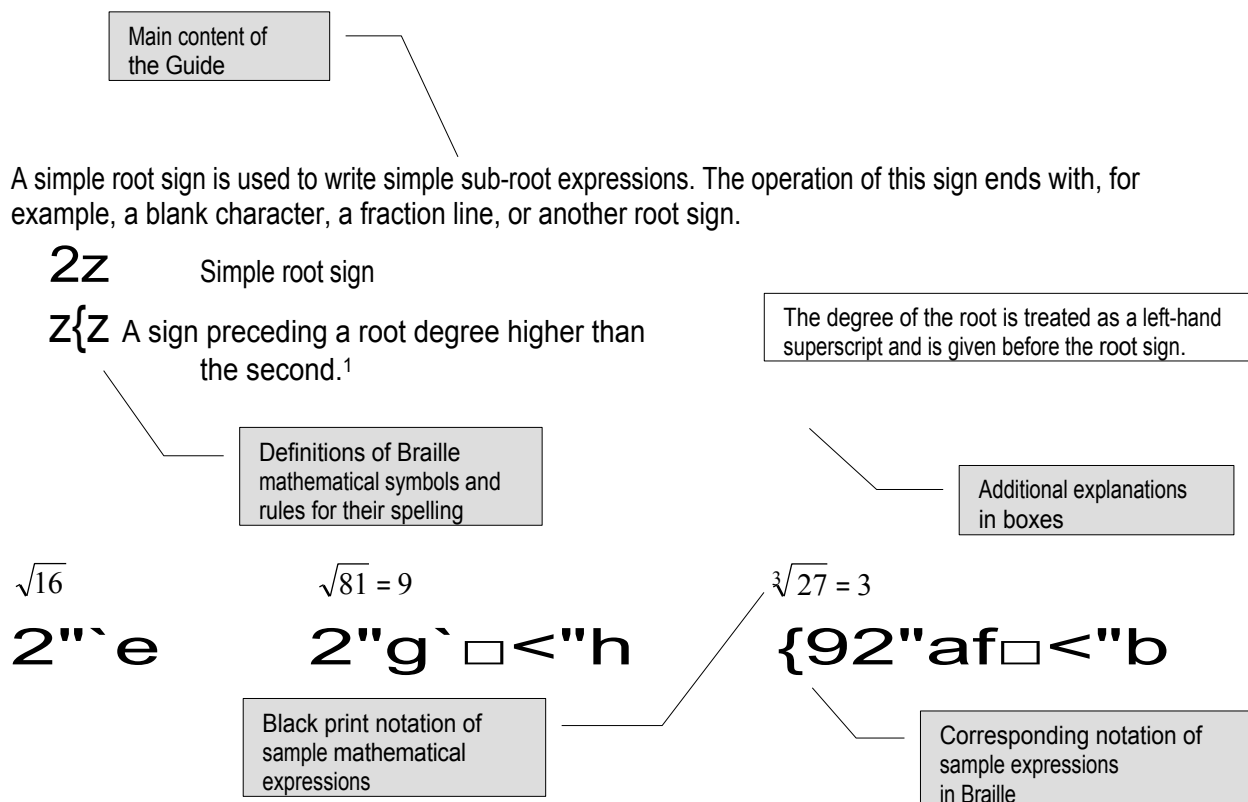
Below are all the characters of the dot writing system as proposed by Louis Braille. The chart includes letters of the Latin alphabet, punctuation marks, and other characters of universal use. In addition, characters assigned to Polish national letters are provided.

a	b	c	d	e	f	g	h	i	j	
@	a	b	c	d	e	f	g	h	i	series 1
k	l	m	n	about	p	q	r	s	t	
j	k	l	m	n	o	p	œ	r	s	series 2
u	v	x	y	z	z			z		
t	u	w	x	y	%	\$	Z	]	\	series 3
ą	ł	ć	ń	ę				ś	w	
0	1	2	3	4	5	6	7	8	v	series 4
,	;	:	/	?	!	()	"	**	"	
0	=	9	-	>	*	<	•	)	(	series 5
			line character	ó	numb er sign	capital letter letters	characte italics			
!	=	{	?	/	"	#	^	0	;	series 6
apostrop he period	hyphen		blank character							
—	,	&	□							

INTRODUCTION

Notations used in the Guide

The ways in which the diverse information in the Guide is presented are shown in the following excerpt.



The definitions of Braille mathematical symbols and spelling rules require more detailed explanation. The limited number of Braille symbols (64) requires precise and consistent application of specific rules in mathematical notation. This is because the same symbol can have multiple meanings. The sequence of characters plays an important role in Braille notation. The absence of a character (space, pause) also conveys particularly important information. In such cases, the notation of expressions very often refers to a so-called "blank character." For this purpose, two special characters are used in the Guide in the definitions of Braille characters:

- ²
- Z orientation sign replacing any Braille sign,
 - $\square$ empty sign (pause, space, no sign).

The orientation sign helps sighted people learn the rules of mathematical Braille notation. This sign is not the so-called "six-dot," marked in the Guide as \$.

The above signs used in definitions clearly define the "neighborhood" of the symbols being defined.

¹ Chapter of the Guide – *Left-hand indicators*.

² Character taken from T. Rynkowski's study – *Mathematics written in Braille*, Owińska 2000. [6]

Sequence of characters³

Due to the position of the "blank character" in mathematical Braille notation, there are 6 groups of characters.

	Definition of a character	Space before the character	Space after the character
Group A	2	YES or NO	Takes into account the definition of character groups A', B', C'.
Group B	□*	YES	
Group C	z0	NO	
Group A'	7	Takes into account the definition of character groups A, B, C.	YES or NO
Group B'	0		YES
Group C'	0z		NO

For clarity, only one character from each group is shown in the above table.

The full definition includes the sequence of characters before and after the defined symbol.

Character definition	Spacing before the character	Spacing after the character
z0z	NO	NO
z5□	NO	YES
□n<z	YES	NO
7	YES or NO	YES or NO
6l□	YES or NO	YES

When writing mathematical expressions, there may be cases where the consistent use of the "blank character" according to the rules given above leads to contradictions. This issue is discussed in detail in the chapter of the Guide entitled "*Mutual Position of Characters*."

³ The symbols in the list are actual mathematical symbols. Their detailed definitions, meanings, and rules of use are discussed in later chapters of the Guide.

NUMBERS

In the Braille system, Arabic numerals are written using the characters of the first series of the alphabet preceded by a *number sign* (number symbol).

"Z – number sign, number symbol

's cardinal numbers

1	2	3	4	5	6	7	8	9	0
"`	"a	"b	"c	"d	"e	"f	"g	"h	"i

nd ordinal numbers

Ordinal numbers are written with a period after the number.

first	second	third	fourth	fifth
"` –	"a–	"b–	"c–	"d–
sixth	seventh	eighth	ninth	zero
"e–"		"g–"		"i–
"f–"		"h–"		"
tenth		twenty-eighth		
"` i–		"ag–		

's numbers

6	68	–682
"e	"eg	,"ega

multi-digit number notation

2,584,537,028	"a–dgc–dbf–iag
2584537028	"adgcdbfiag

4

The notation using the dot character -.

The same number written without a period is less legible.

A blank character (space) must not be used as a separator between groups of digits in multi-digit numbers.

⁴ In Braille, only a comma is used to separate the whole number from the decimal part when writing decimal numbers (see the section on *decimal numbers* in the Guide). Therefore, there is no ambiguity in Braille notation, which sometimes occurs in black print, where a decimal point is sometimes used.

c numbers

In decimal numbers, the whole number is separated from the fractional part by a comma 0.

7.29

"f0ah

0.072

"i0ifa

50.347.296

"di0bcf-ahe

0.333... = 0,(3)

"i0bbb---□<"i01b?

The use of a decimal point in multi-digit numbers, as in the case of whole numbers, improves readability.

Percentages and per mille

Percentages and per mille in Braille are indicated by the following symbols:

z"i(– percent

z"i((– per

mille 0.25 = 25%

48% = 480‰

"i0ad□<"ad"i("cg"i(□<"cgi"i((

Roman notation numbers

Roman notation

Arabic notation

I #h

1

V #u

5

X #w

10

L #k

50

C #b

100

D #c

500

M ##l

1000

Examples of numbers in Roman numerals:

III (3)⁵

#hhh

IV (4)

#hu

XLII (42)

#wkhh

CDLXXXIV (484)

#bckwwwu

MDCCCXXXVII (1837)

#lcbbbwwwuhh

⁵ The equivalents of the numbers in Arabic notation are given in brackets.

IC ALPHABET CHARACTERS

In Braille dot notation, the same characters are used to represent letters from different alphabets. In addition, the same character can represent either a lowercase or uppercase letter. Special key characters called *alphabet characters* enable unambiguous notation.

nd Latin letter notation

The letters of the Latin alphabet are used to write algebraic expressions, units, symbolic representations of physical quantities, chemical reactions, and in many other cases.

The following key characters are used to denote Latin letters:

$\&z$	lowercase Latin letter	
$\#z$	uppercase Latin letter a b	
	w	
$\&`$	$\&a$	$\&v$
A	B	W
$\#@$	$\#a$	$\#v$

Alphabet characters precede the actual letter.

Transcription of Greek letters

The letters of the Greek alphabet are primarily used to denote angles. They can also be found in various branches of mathematics, physics, and chemistry.

The following key symbols are used to denote Greek letters:

$;\mathbf{z}$	lowercase Greek letter symbol	
$^{\wedge}\mathbf{z}$	Greek uppercase letter symbol	
α	β	ω
$;\mathbf{`}$	$;\mathbf{a}$	$;\mathbf{v}$
A	B	Ω
$^{\wedge}\mathbf{@}$	$^{\wedge}\mathbf{a}$	$^{\wedge}\mathbf{v}$

's printed characters highlighted

$\#z$	bold
$^{\wedge}\mathbf{z}$	italics
$;\mathbf{from}$	simple printing
$=\mathbf{from}$	handwriting

The typeface symbol can only be used in combination with an alphabet symbol. It is followed by an alphabet symbol and a letter.

M	P	Σ	F
$^{\wedge}\mathbf{\#L}$	$\mathbf{\#\#O}$	$\mathbf{\#^{\wedge}r}$	$=\mathbf{\#e}$
Capital Latin letter written in italics	Bold Latin capital letter	Bold Greek capital letter	Handwritten capital Latin letter

nd Greek alphabet

	α	A		β	B		γ	Γ
alpha	;	^	beta	;	a	gamma	;	f
	δ	Δ		ϵ	E		ζ	Z
delta	;	c	epsilon	;	d	zeta	;	y
	η	H		θ	Θ		ι	I
eta	;	4	theta	;	3	jota	;	h
	κ	K		λ	Λ		μ	M
kappa	;	j	lambda	;	k	mi	;	l
	ν	N		ξ	Ξ		$\omicron$	O
ni	;	m	ksi	;	w	omikron	;	n
	π	Π		ρ	P		σ	Σ
pi	;	o	ro	;	œ	sigma	;	r
	τ	T		υ	Y		φ	Φ
tau	;	s	ypsilon	;	t	fi	;	e
	χ	X		ψ	Ψ		ω	Ω
chi	;	%	psi	;	x	omega	;	v

Rules for the use of alphabet characters and bold print⁶

Alphabet characters must be used in the following situations in order to eliminate ambiguity in the notation:

1. Before a letter that replaces a mathematical expression and appears separately in mathematical text, e.g.

The numbers a , b , c , and d form an arithmetic sequence.

#khbyax **&`0** **&a0** **&b** **h** **&c**
svnœy0 **bh0f** **œxslsdxby**
x-

The letters a , b , c , and d , which denote specific numbers, are preceded by a lowercase Latin letter.
 The conjunction "and" is not preceded by any key character.

⁶ These rules are not philological in nature, but refer to Braille mathematical notation.

- Before the first letter of a mathematical expression⁷.
- Before the letter of the first series of the Braille alphabet that is a variable in a mathematical expression, so that the letter is not read as another digit.

 $2ac$ **"a&`b**

Omitting the lowercase Latin letter in the expression $2ac$ will cause it to be read as the number 213.

 213 **"a`b**

- Once an alphabet or typeface character⁸ is used in a mathematical expression, it remains valid until the end of that expression or until it is replaced by a character from another alphabet or typeface.

 $41abC$ **"c`&`a#B** $5KLM$ **"d#JKL** $6\alpha\beta\gamma$ **"e;`af ;`a^v** $\alpha\beta\Omega$

Since most texts are written in plain (normal) font, the plain font symbol (as opposed to bold or italic) is usually omitted. If it is necessary to show a typeface other than plain, then the typeface symbol should precede the alphabet symbol. The combination of two symbols (typeface and alphabet) means that the first one is a typeface symbol and the second one is an alphabet symbol.

 $abcd$ **#&`abc** $ABCD$ **^#`abc**

- Alphabet characters can be used redundantly in algebraic expressions and formulas if there is a concern that the type of letter may be misread, e.g., due to the insufficient experience of the recipient of the notation (e.g., a student).

 $\alpha + \beta + \gamma = 180^\circ$ **;;`□\*;a□\*;f□<""`gi(** $W = UIt$ **#v□<#t#h&s** $U = IR$ **#t□<#h#OE**

UNIT NOTATION – BASICS⁹

Units in Braille are preceded by a key unit symbol, also known as *the name sign*.

6Z – name sign, unit sign

 $1m$ 1 km $5\frac{m}{s}$ $230V$ **"`6l****"`6jl****"d6l7r¹⁰****"abi6#U**

⁷ Chapter of the Guide – *Algebraic expressions*.

⁸ Chapters of the Guide – *Roman letters, Greek letters, Bold type characters*.

⁹ Issues related to the notation of units are discussed in the Guide chapter – *Units and operations on units*.

¹⁰ The fraction bar **7** is used in the notation of this unit. This symbol is discussed in the Guide chapter – *Fractions*.

2min

5N

2.5m²

"a6lhm "d6#M "a0d6l/:¹¹

Units written in full are not preceded by a unit symbol.

One meter

1 meter

5 newtons

#ldcdm□ldsOE " "ldsOE "d□mhtsnm/v

The currency symbol is not used with currency names and abbreviations. The currency name or abbreviation should be separated from the number by a space.

2 zlotys

2 zł

2 PLN

"a□y1nsd "a□y1 "a□#O#K#m

5 PLN 50 gr

5.50 PLN

2

"d□y1□ "di□fOE "d0di□y1 "a□!d

RECORDING THE DATE AND TIME

Date format

Dates should be written in accordance with the order of their components in black print, separating the components of the notation with dots, slashes, or hyphens—as in flat print—preceding the first of them with a numerical sign. Upper case numbers (first series) should be used. It is acceptable to write the date with a number preceding each component.

03/15/2002 " "d-ib-aiia or " "d-"ib-"aiia

98/08/26 "hg.ig.ae or "hg."ig."ae

2002-03-15 "aiia,ib,"d or "aiia,"ib," "d

Entries using Roman numerals are also found.¹²

February 15, 2011 " "d□#hh□"ai"

al notation of time

When writing the time, the full hour is written with a cardinal number (characters from the first series). Minutes are written immediately after the period (third dot) or after the colon with characters from the same series (minutes are not preceded by a number).

⁰²⁰

05:40

18.25

"i-ai "id9ci " "g-ad

¹¹ The exponent symbol / is used in the notation. This symbol is discussed in the chapter of the Guide entitled *Powers and Exponents*.

¹² Guide section – *Roman numeral notation*.

OPERATION AND RELATIONSHIP SYMBOLS

signs for operations

+	$\square^*Z$	addition	addition and subtraction
−	$\square,Z$	subtraction	
±	$\square^*,$	plus-minus	
	with		
m	$\square^*,$	minus-plus	multiplication
	with		
·	$\square\text{--with}$	dot in a spaced notation	
·	Z-Z	dot in writing without spacing	
×	$\square'Z$	cross	
:	$\square.Z$	division sign	division
	$\square 7 \square$	fraction bar in full fraction notation	
	z7z	fraction bar in abbreviated fraction notation	

Note:

- Arithmetic symbols in Braille dot notation belong to a group of symbols that must be preceded by a space – an empty character.
- The exception is *the multiplication sign*, which can be written in two ways – with or without a space.
- In Polish mathematical notation, ., is used as the division sign (colon). In the 1986 Marburg notation, the division sign is 9.
- The notation of a fraction line is discussed in detail in the chapter of the Guide entitled *Fractions*.

5 + x	"d□*&w
67 : 14	"ef□."`c
24.6 + 2 – 4.8	"ac0e□*"a□,"c0g
	in the record with a dot with a space
	in the record with a dot without a space
12 · 3	"`a□-"b      "`a-"b

- No space before an operation sign (except for *the multiplication sign*) is a serious error in writing. The

operation 5 + x written incorrectly in Braille without a space "d*&w
means $\frac{5}{6}x$, because the first three characters "d* are the fraction $\frac{5}{6}$.

characters
of the
relation

	$\square < z$	equals
	$\square) < z$	does not equal
$=$	$\square < < z$	equals identity
$\neq$	$\square) < < z$	not equal to identity
$\equiv$		
$\neq$		
$:=$	$\square 9 < z$	equals by definition
$\approx$	$\square > > z$	equals approximately
$>$	$\square n 0 z$	greater than
$\geq$	$\square n < z$	greater than or equal to
$\nlessgtr$	$\square) n 0 z$	not greater
$\gg$	$\square n N 0 z$	significantly greater
$<$	$\square 8 - z$	smaller
$\leq$	$\square 8 < z$	smaller or equal
$\nlessgtr$	$\square) 8 - z$	not smaller
$\ll$	$\square 8 8 - z$	significantly less

$$7 - 4 = 3$$

$$2 \cdot 5 < 47 : 3$$

"f $\square$, "c $\square <$ "b

"a - "d $\square 8 -$ "cf $\square .$ "b

PARENTHESES

Mathematical parentheses

(...)	$1 z --- z ?$	round
[...]	$Z z --- z \backslash$	square
{ ... }	$8 z --- z n$	bracket
$\langle \dots \rangle$	$Z - z --- z \& \backslash$	triangular

$$(14 - 5) + 7 > - [1 + 2(5 + 3)]$$

1 " ` c  $\square$  , "d ?  $\square$  \* "f  $\square n 0$  , Z " ` $\square$ * "a 1 "d $\square$ *
* "b ? \

$$2 + \{4 - [5 + (6 - 2)] + 3(6 + 4)\} = 2 + \{4 - [5 + 4] + 30\}$$

"a $\square$ * 8 "c $\square$, Z "d $\square$ * 1 "e $\square$, "a ? \ $\square$ *
* "b 1 "e $\square$ * "c ? n $\square <$ "a $\square$ * 8 "c $\square$, Z "d $\square$ *
* "c \ $\square$ * "b in

Parentheses for non-mathematical text

Parentheses for non-mathematical text are used for additional explanations. They are sometimes called literary parentheses. In black print, they take the form of round parentheses.

(...) <z---z< literary brackets for non-mathematical text

$$5 - 3 = 2 \text{ (because } 2 + 3 = 5 \text{)}$$

"d□,"b□<"a□<an□"a□*"b□<"d&<

[1]

Unless it leads to misunderstandings, the & sign before the opening or closing parenthesis is omitted.

(...) <z -----z<

However, this sign should not be omitted when the end of the parenthesis occurs immediately after a number.

Omitting the & sign before the closing parenthesis in expression [1] completely changes its meaning.

"d□,"b□<"a□<an□"a□*"b□<"d<

This expression will be read as

$$5 - 3 = 2 \text{ (because } 2 + \frac{5}{3} = \frac{11}{3} \text{)}$$

|...| !Kz---z^□ absolute value, modulus

$$|-5| = 5$$

!K,"d^□<"d

$$-|- (7 + 4) | < | (4 - 7) |$$

,!k,1 "f□*"c?^□8-!k1 "c□,"f?^

DIVISORS NUMBERS

| !K is a divisor

⊄)!K is not a divisor of 5

| 25 (5 is a divisor of 25)

"d□!K"ad or "d!k"ad

$$5 \nmid 27 \text{ (5 is not a divisor of 27)}$$

"d□)!K"af

¹³ The notation of fractions is discussed in the chapter of the Guide – *Fractions*.

SEPARATING NUMBERS FROM PUNCTUATION MARKS

The following punctuation marks

:	9□	colon semicolon
;	:□	question mark
?	>□	
!	*□	exclamation mark
"	(	closed quotation marks
)	<□	closed parenthesis for non-mathematical text ¹⁴
,	0□	comma

belong to a group of characters used in mathematical notation that must be written without any space on the left. A sixth point is placed before the punctuation mark after the number. This rule does not apply to date notation.¹⁵

Correct notation using the & sign

If the & sign is mistakenly omitted

12;	"`a&:	"`a:	$\frac{12}{2}$
12?	"`a&>	"`a>	$\frac{12}{5}$
12!	"`a&*	"`a*	$\frac{12}{6}$

The correct notation of numbers and punctuation marks is very important when giving point coordinates, writing numerical ranges, sets, etc.

$P = (3.5)$	#o□<1"b&0□"d?
$P = (3.2 ; 5.4)$	#o□<1"b0a&:□"d0c?
$X = \{1, 2, 3, 4\}$	w□<8""&0□"a&0□"b&0□"cn
$(3.2 ; 5.4)$	1"b0a&:□"d0c&\
$A = \langle 2, 5 \rangle$	#`□<Z-"a&0□"d?

The use of punctuation marks can cause ambiguity not only in the notation of numbers. The use of the & sign in the following example before the comma immediately after the closing curly bracket eliminates ambiguity – curly bracket followed by a comma, or a majority sign?

$$B = \{2, 5, 6\}, \quad \#a \square < 8 "a \& 0 \square "d \& 0 \square "en \& 0$$

¹⁴ This character has already been discussed above in the section of the Guide entitled *Brackets for non-mathematical text*.

¹⁵ Guide chapter – *Date notation*.

CARRYING OVER PARTS OF MATHEMATICAL EXPRESSIONS TO THE NEXT LINE

When transferring parts of mathematical expressions to the next lines, the following rules apply:

1. Where possible, the part of the mathematical expression that does not fit on the line should be carried over the mathematical operation sign or equal sign, which is placed at the end and repeated at the beginning of the next line.

$$2 + \{4 - [5 + (6 - 2)] + 3(6 + 4)\} = 2 + \{4 - [5 + 4] + 30\}$$

"a□*8"c□,Z"d□*1"e□,"a?\□*
"b1"e□"c?n□<"a□*8"c□,Z"d□*
"c\□"bin

2. A mathematical expression can also be carried over to a place where there is a blank character. In this case, a period is used as the carry character, placing it in the blank box where the formula was interrupted. The & character means that the formula does not end on that line.

$$2 + \{4 - [5 + (6 - 2)] + 3(6 + 4)\} = 2 + \{4 - [5 + 4] + 30\}$$

"a□*8"c□,Z"d□*1"e□,"a?\□*"b1"e&
*"c?n□<"a□*8"c□,Z"d□*"c\□*"bin

3. If a mathematical expression is interrupted at a place where there is no empty box, then the ! sign is used as a carry sign. This sign must be repeated at the beginning of the next line.

$$2 + \{4 - [5 + (6 - 2)] + 3(6 + 4)\} = 2 + \{4 - [5 + 4] + 30\}$$

"a□*8"c□,Z"d□*1"e□,"a?\□*"b1!
!"e□*"c?n□<"a□*8"c□,Z"d□*"c\&
*"BIN

ALGEBRAIC EXPRESSIONS

Appropriate alphabet characters are used to denote variables occurring in algebraic expressions. Braille notation also precisely defines the rules for their use.¹⁶

In addition, the following rules apply to the notation of algebraic expressions:

1. Once an alphabetical character is used in an expression, it remains valid until the end of that expression or until it is replaced by another alphabetical character.
2. The use of an alphabetical symbol is required before the first letter of a mathematical expression. An alphabetical symbol should also be placed before a letter denoting a mathematical expression and appearing separately in mathematical text. This issue is discussed in the chapter of the Guide – *Rules for the use of alphabetical symbols and highlighted print (rule 1)*.

$$3a + b + c \quad [2]$$

"b&` □ * a □ * b

The lowercase Latin letter & before the variable ` in the Braille notation of expression [2] was used only once, in accordance with the first rule.

$$3x + y + z \quad [3]$$

"b&w □ * x □ * y

The expression [3], which is very similar in black print to expression [2], creates different problems in Braille. The variables *a*, *b*, *c* belong to the first series of the Braille alphabet and, when the alphabet sign is omitted, can be in some cases read as numbers. In the case of variables *x*, *y*, *z* this ambiguity does not occur.

Omitting the lowercase Latin letter & before the variable w in expression [3] will not cause ambiguity in reading. However, according to the second rule, this character must be included.

The application of the rules discussed above is illustrated by the following examples

$$3a + 2b + 4c \quad [4]$$

"b&` □ * "a&a □ * "c&b

In the Braille notation of expression [4], it was necessary to use a lowercase Latin letter & before each variable.

$$3x + 2y + 4z \quad [5]$$

"b&w □ * "ax □ * "cy

In the Braille transcription of expression [5], the lowercase Latin letter & is used only once – before the variable w.

$$2aC + 4Bc - 3.2BD$$

"a&` # b □ * "c# a&b □ , "b0 a# ac

$$2abc - 3BCD - BD$$

"a&` ab □ , "b# abc □ , ac

$$2abc - 3BCd - bd$$

"a&` ab □ , "b# ab&c □ , ac

$$2abc - 3a\beta y$$

"a&` ab □ , "b; ` af

¹⁶ Chapter of the Guide – *Alphabet characters*.

SETS

3. Alphabet characters can be used redundantly (e.g., capital letters, Greek lowercase letters) in algebraic expressions and formulas if there is a concern that the type of letter may be misread, e.g., due to the insufficient experience of the recipient of the notation (e.g., a student). Therefore, the following notation is also correct:

$$\begin{array}{ll}
 2aC + 4Bc - 3.2BD & "a\&` \#b\Box * "c\#a\&b\Box, "b0a\#a\#c \\
 2abc - 3BCD - BD & "a\&` ab\Box, "b\#a\#b\#c\Box, \#a\#c \\
 2abc - 3BCd - bd & "a\&` ab\Box, "b\#a\#b\&c\Box, ac \\
 2abc - 3\alpha\beta\gamma & "a\&` ab\Box, "b;` ;a;f
 \end{array}$$

SETS

Designations of sets and sets of numbers

A	$\#$	set A
N	$\#\#M$	set of natural numbers
C	$\#\#b$	set of integers
R	$\#\#\text{OE}$	set of real numbers
W	$\#\#V$	set of rational numbers
IW	$\#\#hv$	set of irrational numbers
¹⁷		
C^+	$\#\#b^*$	set of positive integers
R^-	$\#\#Q,$	set of negative real numbers
$\emptyset$	$\%n$	empty set

Relation and operation symbols on sets

$\in$	$!d$	belongs
$\notin$	$)!d$	should not
$\subset$	$\Box 1-z$	contains, is contained, is a subset of
$\not\subset$	$\Box)1-z$	does not contain, is not included, is not a subset
$\supset$	$?0z$	contains
$\not\supset$	$\Box)?0z$	does not contain

¹⁷ A so-called "mark" was used to designate the collection. This issue is discussed in the chapter of the Guide entitled "So-called 'marks'."

$\cup$	$\square 2-z$	sum of sets
$\cap$	$\square /-z$	product of sets
$\setminus$	$\square 0-z$	difference of sets
$\times$	$\square 'z$	Cartesian product of sets
$\cup$	$\%2z$	indexed sum of sets
$\cap$	$\%/z$	indexed product of sets
$N \subset C \subset W \subset R$	$##m\square 1-##B\square 1-##v\square 1-##\text{œ}$	
$W \not\subset W$	$##V\square)1-##HV$	
$R \supset N$	$##Q?0##M$	
$2 \in C^+$	$"a\square!d###b^*$ or $"a!d###b^*$	
$C = A \cup C$	$B = D \setminus C$	
$#b\square<\#` \square 2-#b$	$#a\square<\#c\square 0-#b$	

In accordance with the rules for using ^{alphabet} characters, the above expressions can also be written as follows:

$C = A \cup C$	$B = D \setminus C$
$#b\square<` \square 2-b$	$#a\square<c\square 0-b$

$U_i \in A_i$

$\%20\&h!d###m4#\`0\&h$ ¹⁹

$\cap_{i \in N} A_i$

$\%/0\&h!d###m4#\`0\&h$

Other symbols used in set algebra

$'$	$z)$	prim ²⁰
$"$	$z))$	bis ²¹
min	"M	minimum

¹⁸ Chapter of the Guide – *Rules for the use of alphabet characters and bold print*, rule 4.

¹⁹ Chapter of the Guide – *Projection technique*

²⁰ The symbol is discussed in the chapter of the Guide – *So-called "marks."*

²¹ As above.

max	"in	maximum
sup	6rto	supremum
inf	6hme	infimum
$\vee$	$\square 20z$	or ²²
$\wedge$	$\square /0z$	and
$\Leftrightarrow$	$\square 0<0z$	if and only if
$\Rightarrow$	$\square <0z$	then (it follows that)

$$B = \{x : x \in \mathbf{N} \text{ and } x < 7\}$$

#a<8&w9w!d##Mh&w8-"fn

$$B = \{x : x \in \mathbf{N} \wedge x < 7\}$$

#a<8&w9w!d##M/0&w8-"fn

$$W = \left\{ x \mid x = \frac{p}{q} \wedge p \in \mathbf{N} \wedge q \in \mathbf{N} \setminus \{0\} \right\}$$

##v<8&w9w<o7p/0o!d##b/0!
!&p!d##b0-8"inn

$$(-\infty; a) = \{x : x \in \mathbf{R} \text{ and } x < a\}$$

1,"\$:&`?<8w9w!d##OEh&w8-`n²³

$$x \in A \setminus B \text{ if and only if } (x \in A \text{ and } x \notin B)$$

&w!d#`0-#avsdcxhskjnvsdcx
fcx1&w!d#`h&w)!d#a?

$$x \in A \setminus B \Leftrightarrow (x \in A \wedge x \notin B)$$

&w!d#`0-#a0<01&w!d#`/0!
!&w)!d#a?

$$A = \{x : x \in \mathbf{C} \text{ and } 0 < x < 5\}$$

#`<8&w9w!d##bh"i8-w8-"dn

²² The symbol is discussed in the chapter of the Guide – *Mathematical Logic*.

²³ The fraction bar symbol 7 used in the dependency is discussed in the Guide section – *Fractions*.

$$(a, b) \in A \times B \Leftrightarrow (a \in A \wedge b \in B)$$

1&`0□a?□!d#`□'#a□0<01&`□!d#`&
/0&a□!d#a?

$$A \subset X$$

#`□1-#w

$$A' = X \setminus A$$

#`)□<#w□0-#` complement of set A to set X

$$\text{If } X = \{1, 2, 3, 4, 5, 6\},$$

#id%dkh□#w□<8""&0□"a&0□"b&0&
"c&0□"d&0□"en&0²⁵

$$A \subset X \text{ and } B \subset X,$$

#`□1-#W□h□#a□1-#W0

$$A = \{1, 2, 3, 4, 5\} \text{ and } B = \{4, 5\}, \text{ then:}$$

#`□<8""&0□"a&0□"b&0□"c&0□"dN
h□#A□<8"c&0□"dn&0□sn9

$$A \cup B = \{1, 2, 3, 4, 5\},$$

#`□2-#a□<8""&0□"a&0□"b&0□"c&0&
"dn&0

$$(A \cup B)' = \{6\},$$

1#`□2-#a?)□<8"en&0

$$(A \setminus B)' = \{4, 5, 6\},$$

1#`□0-#a?)□<8"c&0□"d&0□"en&0

$$B \setminus A = \emptyset,$$

#a□0-#`□<%n&0

$$A \cap B = \{4, 5\}.$$

#`□/-#a□<8"c&0□"dn-

²⁵ The example uses the & character. Explanation in the chapters of the Guide – *Moving part of a mathematical expression to the next line* and *Separating numbers and punctuation marks*.

Graphical representation of intervals

In mathematics lessons, it is often necessary to represent an interval graphically. In flat printing, intervals are marked as rectangles extending over the relevant part of the number line, and the types of endpoints are marked with circles. An empty circle means that the endpoint does not belong to the interval, while a filled circle means that the endpoint belongs to the interval.

Example:

$$A = [2, 5)$$



Using Braille characters, such an interval can be represented graphically as follows:

"a000"d

□\$<<<<w

The first line of the notation corresponds to the number axis, while the second line contains a graphical representation of the interval, where the \$ sign denotes an endpoint belonging to the interval (equivalent to a solid circle), and the symbol denotes an endpoint not belonging to the interval (equivalent to an empty circle in a flat diagram). It is best to place three 0 symbols between the numbers on the number line. The end symbols should be placed as centrally as possible under the corresponding numbers on the number line. If the interval is unbounded, no symbol is written on the number line on the infinity side, nor is the endpoint marked in any way.

Examples:

$$(-\infty, 5)$$

000"d

<<<<w

$$\langle 10, +\infty)$$

"`i0000

□\$<<<<

If it is necessary to place two or more intervals on a single drawing, we place their graphical representations in separate lines, below the number axis:

"a000"b000"d000"``

□\$<<<<<<<<<w (2,5)

□□□□□□w<<<<<<<<<\$ (3,11)

Letter designations for intervals are written on the same line as the graphical representation of the interval (before or after it), with at least one character spacing.

Example:

00,"b000"i000"``f

#`□w<<<<<<<<<<w (-3.17)

<<<<<<<<<\$□#a $(-\infty, 0)$

The graphical representation of intervals allows students to easily locate the area where intervals overlap (the so-called intersection, product), and determine the sum and difference of these sets. An important advantage of this notation is that it can be quickly performed by the student themselves, e.g., when solving problems on a test.

PROJECTION TECHNIQUE

Braille has specific features that distinguish it from many other ways of recording information.

- Braille is read with the fingers, and the sense of touch, unlike the sense of sight, allows only for a point-by-point (partial) reading of an expression.
- Mathematical expressions are written in a linear manner (sequence of characters), limited to 64 characters.
- It is not possible to differentiate between character sizes or use rich mathematical symbols.

Advances in knowledge and new technologies meant that *the International Braille Mathematical Notation for the Blind*, developed in 1955 in Marburg by a team led by Helmut Epheser, needed to be changed. These changes were introduced under the direction of Helmut Epheser in 1986 in Braille and then in 1992 in black print. In order to simplify the notation of complex mathematical expressions as much as possible and at the same time improve their readability, the way they are written was changed by developing a projection technique.

Principles of the projection technique

Projection is the transformation of a two-dimensional notation into a linear one. Formulas containing symbols that replace multi-level black print notation (e.g., fractions, indices, exponents, or roots) are called projectors.

There are three types of projectors: simple, complex, and detailed.

nd simple projectors

They begin with a simple power exponent sign /, a simple upper [or lower 0 index sign, or a simple square root sign 2. These signs are followed by simple expressions containing characters in a single row, i.e., they do not contain other projectors or fractions (except for ordinary ones). There can be no empty characters inside simple projectors. Places that must remain empty according to the rules of notation should be filled with the character !.

The operation of each of these projectors ends with:

- a) with an empty character,
- b) by a character closing a simple projector 4, or 04 or #4, respectively, if the simple projector is part of a complex or detailed projector,
- c) by another projector character,
- d) by a fraction line or fraction end character,
- e) by a closing parenthesis, when the corresponding opening parenthesis precedes the projector,
- f) by exponents and indices that are natural numbers written after the key character as subscript numbers.

$$a^{n+m} = a^n \cdot a^m$$

$$\&\text{'}/m!*l\Box<\text{'}/m4-\text{'}/l$$

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

$$\&\text{'}0m\Box<:\text{'}0m!,\text{'"}4!*'\text{'}0m!*'\text{'"}7:$$

$$(a_k)^2 = a_k \cdot a_k$$

$$\&\text{'}0j/:\Box<\text{'}0j\Box-\text{'}0j$$

$$\frac{\sqrt[3]{3}}{2} = \frac{1}{2} \cdot \sqrt[3]{3}$$

2"b7:□<"`:-2"b

$$(3+a_{n+2})^{.5}$$

1"b□*&`0m!"a?-"d

$$f_n(x) = nx$$

&e0m41w?□<mw

-type projectors

They are started with a compound power exponent sign 0/, a compound upper index sign 0[or lower index sign 00, or a compound root sign 02.

They may contain simple projectors and fractions with no empty characters. The operation of a compound projector ends with:

- an empty character,
- the end mark for a compound projector 04, if necessary also by #4, if the compound projector is part of a detailed projector,
- by another composite or detailed projector symbol,
- by a closing parenthesis, if the corresponding opening parenthesis is before the projector sign,
- by a fraction end mark, if the fraction line is before the projector sign.

There must be no empty spaces in compound projectors either. Where necessary, a filling point ! is used. If ambiguity in the notation is excluded, the 0 character announcing the compound projector may be omitted.

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

02&w7x□<2w72x

$$u_n = \sqrt[n]{3^n + 2^n}$$

&t0m□<[m02"b/m4!"a/m

$$e^{\frac{x}{2}} = \sqrt[x]{e}$$

&d0/w7:□<02d/w

detailed projectors

They begin with simple projector designations preceded by the # sign. They can contain any expressions, other projectors, and empty characters. At the end, specific projector ending characters must be used

#4

$$e^{\sqrt[3]{x+1+2}^x}$$

&d#[92w!"`□*"a#4-d/w

$$u = \sqrt[3]{x^2 + \sqrt{x^2 + 1} +}$$

&t□<^8[9#2w/:□*02w/;!*"`□*"g#4

Braille writing does not contain row change characters. They are marked by the start and end characters of the projector. The start character of the projector raises the row, while the end character lowers it.

1. The base (zero) row includes every character in black print at the line level without any additional characters (indicators, exponents). Expressions may contain common fractions.

$$3x-7 \quad \frac{2^x + \frac{1}{3}}{\frac{2}{3}} \quad 2\pi r \quad "b\&w\Box, "f \quad "a9\&w\Box * " "a9 \quad "a;o\&O\text{E}$$

2. In the nth row (n = 1, 2, ...) there are
 - a) Additions to exponent characters if the base of the exponent is in row (n-1).
 $(x+y)^2$

1&w\Box *x?/: The base of the power is in row 0, the exponent of the power is in row 1.
 $e^{\frac{x}{2}}$

&d0/w7: The base of the power is in row 0, the fraction bar is in row 1, and the numerator and denominator are in row 2.

- b) Additions to the index characters, referring to formulas from row (n-1).

a_{ij}
&`00h0i a is in row zero, index i in row 1, index j in row 2.

- c) The numerator and denominator of a fraction whose fraction line is in row (n-1).

$1 + \frac{b}{a}$
 $1 - \frac{b}{a}$
: " \Box * : &a7` \Box 7 \Box " \Box , : a7` ; The long fraction bar is in row 0., 1+, 1- and the fraction bars in the numerator and denominator are in row 1., variables a and b are in row 2.

- d) Components of formulas that are marked with the root symbol, if it is located in the (n-1) row.

$\sqrt{\frac{x}{y}}$
02&w7x The complex root symbol is in row 0, the fraction bar is in row 1, and the variables x and y are in row 2.

In addition, the following rules apply:

1. An additional sign (e.g., power, index) is in the row of the symbol to which it is attached. It indicates an increase in the row for the expression immediately following it.
2. The base symbol causes an increase in the row for the expression immediately following it.
3. The fraction line indicates that the expression in the numerator (before the fraction line) and in the denominator (after the fraction line) are in a higher row than the fraction line.
4. The end mark of the projection is in the same row as the corresponding start mark.

There are two methods for writing fractions:

- The use of abbreviated notation saves a considerable amount of space.

The following symbols are used in the standard notation of fractions:

Z: fraction end sign

- $$\frac{2x + 3y - 4}{4x - 5y}$$

```
:"a&w□*"bx□,"c□7□"cw□,"dx:
```

In addition, the following rules apply:

If the numerator and denominator do not contain a space, the fraction start and end symbols and spaces on both sides of the fraction bar are omitted in the fraction notation.

$$\frac{x}{y} \quad \&w7x$$

```
:&w□7□x;
```

If the denominator is a natural number, it can be written in lower case, without a number sign, immediately after the fraction line.

$$\frac{3x}{4}$$
$$\frac{2a}{7}$$
$$\frac{ab + cd}{4}$$

"b&w7. ☐

"a&`7<□ :&`a□*bc□7.□

If the numerator and denominator are natural numbers, the fraction bar can be omitted and the denominator can be written in a lowered position. This is how it is written:

$\frac{1}{2}$	$\frac{3}{14}$	$\frac{17}{5}$	$\frac{138}{43}$
"`:	"b0.	"`f>	"`bq.9

d mixed numbers

$$2\frac{3}{4} \qquad 4\frac{7}{15} \qquad \frac{1214}{17}$$

a) The denominator is written immediately after the fraction line (without a space), and the empty characters are replaced with a filler character !. The fraction end character is omitted, but it is mandatory to leave an empty character after the denominator. The same applies to the numerator.

abbreviated notation	full notation
$\frac{x+y}{x-y}$ $\&w!*x7w!,x$	$\frac{x+y}{x-y}$ $:\&w\square*x\square7\square w\square,x;$

b) The fraction sign can only be omitted if the numerator is short. It cannot be omitted if the numerator begins with a minus sign.

$$\frac{-p-q}{n} : , \& o ! , p 7 m$$

If the denominator begins with a sign that requires an empty space on the left (e.g., a minus sign), it is replaced with the sign !.

$$\frac{p+q}{-n}$$

The methods described in points a) and b) can only be used if the numerator and denominator do not contain any fractions other than ordinary ones.

c) If the numerator ends with a character that requires a space after it, a filler character ! is placed between it and the fraction line.

$$\frac{(n+1)!}{2n}$$

The factorial sign requires a space of 5□ after it. In abbreviated notation, no space is placed before the fraction line, hence this space is filled with the sign !.

²⁶ The factorial sign used in the fraction is discussed in the chapter of the Guide – *Probability and Combinatorics*.

Examples of operations on fractions ordinary

$$\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = \frac{3}{2} = \frac{11}{2}$$

"b.□*"b.□<"e.□<"b:□<"`":

$$2.6 - \frac{6}{15} = 2 \frac{18}{30} - \frac{12}{30} = 2 \frac{6}{30} = 2 \frac{1}{5} = 2.2$$

"a0e□,"e0>□<"a" `g9(□,"`a9(□<
<"a"e9(□<"a" `>□<"a0a

The multiplication sign – a dot – can be used with or without a space.²⁷ The notation with a dot without a space is shorter. According to some, the use of a space improves the readability of the formula.

$$4 \frac{3}{4} (1.2 - 0.8) = 4 \frac{3}{4} \cdot 0.4 = \frac{19 \cdot 4}{4 \cdot 10} = \frac{19}{10} = 1.9$$

Braille notation with a dot without spacing.

"c"b.-1" `0a□,"i0g?□<"c"b.-"i0c□<
<"`h.-"c0(□<"`h0(□<"`0h

$$4 \frac{3}{4} (1.2 - 0.8) = 4 \frac{3}{4} \cdot 0.4 = \frac{19 \cdot 4}{4 \cdot 10} = \frac{19}{10} = 1.9$$

Braille notation with a dot with a space.

"c"b.□-1" `0a□,"i0g?□<"c"b.□-
-"i0c□<"`h.□-"c0(□<"`h0(□<"`0h

Algebraic fractions

The rules and principles for writing algebraic expressions apply to the notation of algebraic fractions.²⁸

$$\frac{3a+b}{c} \quad "b\&`!*a7b\Box$$

$$\frac{3x+y}{z} \quad "b\&w!*x7y\Box$$

- Once an alphabetical character is used in an expression, it remains in effect until the end of that expression or until it is replaced by another alphabetical character.
- The use of an alphabetical character applies before the first letter of a mathematical expression.
- Capital letters and Greek letters, both upper and lower case, may be used redundantly.

When the numerator is long, a fraction mark should be placed.

$$\frac{3a+2b}{4c} \quad : "b\&`!* "a\&a7 "c\&b\Box$$

$$\frac{0.6a+1.4b}{5a-6b} \quad : "i0e\&`*" `0c\&a7 "d\&`,`e\&a\Box$$

²⁷ Chapter of the Guide – Operation signs.

²⁸ Chapter of the Guide – Algebraic expressions.

$$\gamma = \frac{\alpha + \beta}{2} = \frac{\alpha}{2} + \frac{\beta}{2}$$

;f□<;`!\*;a7:□<;`7:□*;a7:

$$2\frac{2}{3} + \frac{3.2p - 1.6q}{1\frac{2}{5} + \frac{1}{2}r}$$

"a"a9□*:"b0a&o!,"`0ep7
7""a>!*"" :œ□

$$2\frac{2}{3} + 3p - \frac{1.6q}{1\frac{2}{5} + \frac{1}{2}r}$$

"a"a9□*"b&o□,"`0ep7""a>!*"" :œ□

$$\frac{\frac{2}{3} + 3.2p - 1.6q}{1\frac{2}{5}} + \frac{1}{2}r$$

:"a9!*""b0a&o!,"`0ep7""a>□*"":œ

$$\frac{\frac{2}{3} + 3.2p - 1.6q + \frac{1}{2}r}{1\frac{2}{5}}$$

:"a9!*""b0a&o!,"`0ep!*"" :œ7""a>

$$\frac{\frac{2}{3}x - 1.5y}{2y + \frac{3}{8}z} \quad [6]$$

:"a9&w□,"`0dx□7□"ax□*
*"b'y;

Expression [6] written in full form – using fraction start and end symbols.

:"a9&w!,"`0dx7"ax!*""b'y□

Expression [6] written in abbreviated form.

Parentheses – is the notation abbreviated?

In mathematical Braille notation, algebraic expressions can also be written unambiguously using parentheses. This issue is illustrated by the following examples.

$$-\frac{x-y}{2} = \frac{-(x-y)}{2} = \frac{-x+y}{2} = \frac{y-x}{2}$$

,: &w!, x7: □ <: , 1w□, x?7: □ <
<: , w!*x7: □ < x!, w7:

The above expression can be written unambiguously using parentheses.

$$-\frac{\left[\frac{(x-y)}{2}\right]}{\left[\frac{-(x-y)}{2}\right]} = \frac{-(x-y)}{2} = \frac{(-x+y)}{2} = \frac{(y-x)}{2}$$

, Z1 &w□, x?7: \□ < , 1w□, x?7: □ < 1, w□*
*x?7: □ < 1x□, w?7:

The introduction of parentheses:

- is an interference with the mathematical source text,
- lengthens the Braille notation,
- and, in the case of multiple parentheses for complex expressions, makes it difficult to read.

The use of the multiplication sign – a dot with or without a space – requires additional explanation. A blank character before the multiplication sign ends the projection. In some expressions, this affects the variation in notation, the order of which can also be determined by introducing additional parentheses.

$$\frac{2x}{3yz}$$

"a &w7"bx-y

Multiplication sign – a dot in notation without a space.

$$\frac{2x}{3y}z$$

"a &w7"bx□-y

Multiplication sign – a dot in a notation with a space.

$$e^{xy}$$

&d/w-x

$$e^x y$$

&d/w□-x

An empty character before the multiplication sign terminates a simple projector.

nd compound fractions

It is recommended to write such fractions as follows: the basic fraction in full form, and the fractions in the numerator or denominator in reduced form.

$$\frac{a+b}{a-b} = \frac{1+\frac{b}{a}}{1-\frac{b}{a}}$$

&!*a7`!,a□<:"`□*:a7!□7□"`,
,:a7`;

$$\frac{\frac{a}{3} - \frac{b}{4}}{\frac{x}{x+y}}$$

::&`79□,:a7.□7□;w7w!*x;

$$\frac{x+\frac{x-1}{y-1}}{x-\frac{x+1}{y+1}} = \frac{\frac{x(y-1)+(x-1)}{y-1}}{\frac{x(y+1)-(x+1)}{y+1}} = \frac{\frac{xy-x+x-1}{y-1}}{\frac{xy+x-x-1}{y+1}} = \frac{y+1}{y-1}$$

:&w□*:w!,"`7x!,"`□7□w□,:w!*"7
7x!*"!;□<::w1x□,"`?!\*1w□,"`?7x!,
, "`□7□:w1x□\*"`?!,1w□*"`?7x!*" ;&
<::wx!,w!*w!,"`7x!,"`□7□:wx!*
*w!,w!,"`7x!\*" ;□<x!\*"7x!,"`

$$\frac{2\frac{2}{3} + \frac{1.2-0.7}{2.6}}{\frac{1.2+0.6}{0.8} - 3\frac{4}{5}}$$

:"a"a9□*:"`0a!,"i0f7"a0e□7
7□:"`0a!*"i0e7"i0g□,"b"c>;

$$\frac{1}{2} + \frac{1}{2\cdot 3} + \frac{1}{2\cdot 3\cdot 4}$$

"`:□*"7"a-"b□*"7"a-"b-"c□

²⁹ Chapter of the Guide – Full notation of fractions.

³⁰ Chapter of the Guide – Abbreviated notation of fractions.

POWERS AND S INDICES

Key symbols for exponents and right-hand side

z/z Straight power exponent sign

$z0z$ simple subscript sign

$z\{z$ simple superscript sign

In Braille, the exponent and right-hand subscripts are written after the base symbol, each preceded by the appropriate key character. The exponent sign, subscript sign, and superscript sign begin a straight projector.

- Indices and exponents that are integers are written after the appropriate key character in subscript without a number sign. At the same time, they end the projector.

 a^{-3} $\&\text{`O}.9$  $a^0$  $\&\text{`O}(\$ a^{12} $\&\text{`OO}:$  $b^{-1}$  $\&a[,0$  $b^4$  $\&a[.$  $b^{31}$  $\&a[90$  $x^{-3}$  $\&w/,9$  $5^4$  $"d/.$  $b^{10}$  $\&a/0(\$  $a^{11}a^{22} \cdot a^{12}a^{21}$  $\&\text{`OOO}\text{`O}::\square,\text{`OO}:\text{`O}:0$  $a^2b^3c$  $2$  $\&\text{`/:}a/9b/:$ $2x^2\sqrt{y}$ $"a\&w/:2x$ $\frac{x^3}{3y}$ $\&w/97"bx$ $\left(\frac{1}{3}\right)^4$ $a^{\frac{1}{2}}$ $\&\text{`/}"\text{`}:$ $8^{-3} = \frac{1}{8^3}$ $"g/,9\square<"\text{`7}g/9$

- A simple projector can be an exponent or a right-hand index. It contains characters in the same row (it does not contain other projectors or fractions other than ordinary ones). There can be no empty characters inside projectors. A simple projector ends with:

a) the nearest empty character

 $x^n + 9$ $\&w/m\square*"h$ $y^{2n-3} \cdot z$ $\&x/"am!,"b\square-y$ a^{n+2} $\&\text{`Om}\square*"a$  $a^{n+1} - 5$  $\&\text{`Om}!"*\text{`}\square,"d$

- b) end of simple projector 4, by a pair of characters 04, or #4, when the simple projector is part of a more complex projector

$$\begin{array}{c} x_i \cdot x_j \\ y^{2n-3} \cdot z \\ \&w0h4-w0i \&x/"am!,"b4-y \\ f_n(x) \quad f_{n+1}(x) \quad g_{ij}(y) \\ \&e0m41w? \&e0m!*"\`41w? \&f0hi41x? \end{array}$$

- c) by another character starting another projector

$$\begin{array}{c} (a_n)^k \quad (P_{2n-1})^m \\ \&`\&O m/j \quad \#o0"a\&m!,"`/l \end{array}$$

- d) by a fraction bar or a fraction ending character

$$\begin{array}{c} \frac{x^n}{n!} \quad \frac{1}{4 + x_{n+1}} \\ \&w/m7m5 \ "`7"c!* \&w0m!*"\`; \end{array}$$

- e) by a closing parenthesis when the corresponding opening parenthesis is in a different row.

$$\begin{array}{c} (2 + x_n + 5)^2 \\ 1"a\Box * \&w0m!*"\&d?/: \end{array}$$

3. The exponent or index can be a compound projector. A compound projector can contain simple projectors and fractions with no empty spaces inside. It is opened by the appropriate key character preceded by the compound projector character 0.

z0/z compound power exponent sign

z00z compound subscript sign

z0{z compound superscript character

The compound projector ends:

- the nearest empty character,
- a pair of characters ending the compound projection 04, or #4 if the compound projector is part of a detailed projector,
- by a character starting another compound or detailed projector,
- by a closing parenthesis, when the corresponding opening parenthesis is in a different row,
- by fraction end markings, if the fraction bar is before the projector character.

$$\begin{array}{c} f_{n_k}(x) \\ \&e00\&m0j041w? \end{array}$$

$P_1, P_4, P_9, \dots, P_{n^2}$
 $\#00\&0\#0.\&0\#0)\&0\text{---}0\&$
 $\#00\&m/:04$
 $x_{n_1}, x_{n_2}, x_{n_4}, \dots, x_{n_{2^k}}$
 $\&w00m0004\&0\&w00m0:04\&0\&w00m0.04!$
 $!\&0\text{---}0\&w\#0m00"a/j\#4$
 $A_1 = a, A_2 = a^7, A_3 = a^{31}, \dots, A_k = a^{2^{2k-1}-1}$
 $\#`00\<\&`0\#`0:\<\&`/\<\&0\#`09\<$
 $\<\&`/90\&0\text{---}0\#`0\&j\<\&`0/"a/!$
 $!"aj!,``4!,``04$
 $\frac{x^2}{e^2 \sqrt{2\pi}}$
 $:\&d0/!:w/:7:0472"a;o$ or
 $:\&d0/!:w/:7:04\&7\&2"a;o;$

4. The exponent or index can be a detailed projector. A detailed projector can contain any expressions, other projectors, and empty spaces. It is opened by the appropriate key character preceded by the detailed projector character $\#$. At the end, the characters $\#4$ must be used and cannot be omitted.

 $z\#/z$ detailed power exponent character

 $z\#0z$ detailed lower index character $z\#\{z$
 $\}$ detailed upper index character

 $x_{n1}, x_{n2}, x_{n4}, \dots, x_{n_{2^k}}$
 $\&w0/m00\&0\&w0/m0:\&0\&w0/m0.\&0\&$
 $\text{---}0\&w\#/m00"a/j\#4$

The order of right-hand side indices and the exponent

When a symbol has more than one simple or compound index on the right-hand side, they are written one after the other, preceded by the appropriate key character. Furthermore, if the main symbol is a power expression, the exponent should be placed at the end after all the indices.

$$(x_n^i)^r$$

$$(x_n^i)^r$$

$$x_{n_i}^r$$

$$(P_{a_i}^{ak})^n$$

$$x_{n_i}^r$$

$$(P_{a_i}^{ak})^n$$

$$x_{n_i}^r \quad (P_{a_i}^{ak})^n$$

So-called "marks"

The term "marks" refers to symbols written above or below the main symbol on the right-hand side, or above or below the main symbol, which are not indicators. Certain geometric symbols, the previously mentioned operation symbols, and other symbols may be used as marks.

1. Marks written above or below the main symbol on the right-hand side

$$' \quad z) \quad \text{prim}$$

$$" \quad z)) \quad \text{bis}$$

$$* \quad z. \quad \text{asterisk}$$

$$x \quad z' \quad \text{cross}$$

$$\rightarrow \quad z90 \quad \text{right arrow}$$

$$\leftarrow \quad z09 \quad \text{left arrow}$$

$$+ \quad z^* \quad \text{plus}$$

$$- \quad z, \quad \text{minus}$$

$$\neg \quad z3 \square \quad \text{hook}$$

$$\circ \quad z(\quad \text{circle}$$

2. Symbols written above or below the main symbol

$$\sim \quad z> \quad \text{falka}$$

$$- \quad z9 \quad \text{dash}$$

$$\circ \quad z(\quad \text{circle}$$

$$\wedge \quad z/ \quad \text{roof}$$

$$\cdot \quad z: \quad \text{dot}$$

$$.. \quad z::: \quad \text{two dots}$$

Symbols are written after the symbol to which they refer. They are placed in the same line as the main symbol. The position of the symbol relative to the main symbol is determined by the appropriate key character between it and the symbol. The key characters used for symbols are as follows:

$$z=z \quad \text{upper mark key}$$

$$z;z \quad \text{lower mark key}$$

Lower marks must always be written with a key character, while for upper marks, if there are no conflicts, it can be omitted.

Examples

Full notation

 R^+ $##\text{OE}^*$ A'' $\#` = ))$  $C_-$  $\&B; ,$  $\hat{C}$  $\#b = /$  $\sim A$  $\#` = >$ $b^{\rightarrow}$ $\&a90$ $AB^{\rightarrow}$ $\#` a90$

Abbreviated notation

 R^+ $##\text{OE}^*$ A'' $\#` ))$  $\hat{C}$  $\#b /$  $\sim A$  $\#` >$ $b^{\rightarrow}$ $\&a90$ $AB^{\rightarrow}$ $\#` a90$

When a symbol has both a superscript and a subscript or exponent, in Braille the superscript is usually written immediately after the symbol, before the subscript or exponent.

 $\dot{x}_{n+1}$ a_n'' v^{-2} $\&w:0m!*''`$  $\&`))0m$ $\&u9/:$

It is also possible to use the reverse notation, in which the symbol is written after the index or exponent. However, in this case, it is necessary to show the end of the index or exponent using the end of line symbol 4.

 y_1' x_{n+1}'' $\&x004)$ $\&w0m!*''`4))$

Horizontal brackets

a sign written above the expression

 $\rightarrow \#90z$ $\leftarrow \#09z$ $— \#9z$ $\frown \#1z$

a sign written below the expression

 $\rightarrow ^90z$ $\leftarrow ^09z$ $— ^9z$ $\frown ^1z$

long arrow to the right

long arrow to the left

long dash

long arc

symbol written above the expression

sign written below an expression

 $\overset{\sim}{\#}8$
 $\underset{\sim}{^}8$

long horizontal

bracket Parentheses are always written before the part of the expression that is to be highlighted.

 $\overset{\rightarrow}{AB}$
 $\overline{CD}$
 $\overline{KLM}$
 $\overbrace{A_1 B_1 C_1}$
 $\#90\#`a \quad \#9\#bc \quad \#1\#jkl \quad ^1\#`00a00b00$
Left-hand indicators

Key symbols for left-hand indicators

full notation

abbreviated notation

 $"\{z$
 $\square\{z$

- upper left-hand indicator symbol

 $"Oz$
 $\square Oz$

- left-hand subscript

1. Abbreviated notation should be used in records. Full notation of left-hand indicators should be used if abbreviated notation is ambiguous.
2. The left-hand index sign precedes the main symbol to which it refers. The left-hand index sign begins a simple projector. The index usually ends with the end of projector sign. It can be omitted when the index is a number written in subscript or when the index is followed by another index sign.
3. If the left-hand index is a compound or detailed projector, we use compound index signs or detailed index signs (analogous to right-hand indexes). We end the index with the appropriate projector end signs.
4. If a symbol has a left-hand upper and lower index, they are written one after the other. Each is introduced by the appropriate sign. There is no rule regarding the order in which indices are written.
5. In Braille notation, the degree of a root is treated as an upper left-hand index. It is not necessary to use an index ending sign.
6. The bases of logarithms, which are written in black print as a subscript after the logarithm sign, should be written in Braille as left-hand superscripts.

 $\overset{j}{\underset{n}{\underset{d}{a}}}$
 $O\&h\{i4I$

or

 $[\&i0h4I$

or

 $"O\&h\{i4I$
 $\sqrt[3]{8} = 2$
 $\sqrt[n]{x}$
 $\sqrt[n+1]{y}$
 $\{92" g \square < "a \quad [\&m2w$
 $[\&m!*""`2x$
 $\log_2 8 = 3$
 $\{ :5k" g \square < "b$

ELEMENTS

Among roots, there are simple, compound, and specific roots. These names correspond to the types of projectors with which they are associated.

Z{Z The sign preceding a root degree higher than the second.³¹

The degree of the root is treated as a left-hand superscript and is given before the root sign.

Simple root sign

A simple root sign begins a simple projection. Therefore, an expression marked by a simple root sign can only contain characters that are in the same row (it does not contain other projectors or fractions other than ordinary ones). There can be no empty characters inside the expression.

$$2z \quad \text{Simple root sign}$$

$$\sqrt{16} \quad \sqrt{81} = 9 \quad \sqrt[3]{27} = 3$$

2" e 2" g` □ < " h {92" a f □ < " b

$$\sqrt{x + \frac{1}{2}} \quad 3\sqrt{2x}$$

2&w!*"" : "b2"a&w

The operation of a simple root sign ends:

a) the nearest empty character,

$$\sqrt{x+y} \quad \sqrt{x+y} + \sqrt{x-y}$$

2&w □ * x 2&w!*x □ * 2w!, x

b) projector ending character 4 or pair of projection ending characters 04, or #4, when the simple projector is part of a more complex projector,

$$\sqrt{x+y} e^z \quad \sqrt{x+y} \sqrt[4]{-y}$$

2&w!*x4-d/y 2&w!*x4-2w!, x

$$3\sqrt{2}x \quad 0.5\sqrt{x} \frac{x+y}{x-y}$$

"b2"a4&w "i0d2&w4:w □ * x □ 7 □ w □ , x;

c) character starting the next projector,

$$\sqrt{ab} = \sqrt{a} \sqrt{b} \quad a\sqrt{2a} \sqrt{3b}$$

2&`a □ < 2`2a &`2"a&`2"b&a

³¹ Chapter of the Guide – *Left-hand indicators*.

d) closing parenthesis when the corresponding opening parenthesis is in a different row,

$$(a + \sqrt{b})^2 - b$$

1&`□\*2`a?/:□,a

e) fraction line or fraction ending sign.

$$\frac{\sqrt{2}}{2} \approx 0.7071$$

2"a7"a□>>"iOfif`

nd root complex sign

A compound root sign is obtained by placing a compound projector sign in front of a simple root sign. Simple projectors and fractions can be placed under the root. There can be no empty signs.

02z Composite root sign

The end of a compound root is determined according to the rules for the end of a compound projection.

$$\sqrt{2}\sqrt{} = \sqrt{2} \sqrt[4]{}$$

02"a2"a□<2"a□- {.2"a

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

02&`7a□<2`72a

$$\sqrt{x+y-2}\sqrt{xy}$$

02&w!*x!,"a2wx

$$\sqrt{2x + \sqrt{x-2} - 3}$$

02"a&w!*2w!,"a4!,"b

Detailed root sign

A detailed radical sign is obtained by placing a detailed projector sign in front of a simple radical sign. Any expressions, other projectors, and empty signs can be placed under the radical.

#2z Detailed root sign

At the end of a detailed root, there must be a pair of #4 characters, in accordance with the rules for the end of a detailed projector.

$$\sqrt{2x\sqrt{x+\sqrt{2-x}}+5}$$

#2"a&w02w!*2"a!,w04□*"d#4

$$\sqrt{x+y-2\sqrt{x+y}}$$

#2&w□*x□,"a2w!*x#4

$$\sqrt{x+y-2\sqrt{x+y}+\sqrt{x}}$$

#2&w□*x□,"a2w!*x□*2w#4

$$\sqrt{\sqrt{\sqrt{\frac{1}{8}-\frac{1}{8}-\frac{1}{8}}}}$$

#2022"``'4!,"``'□,"``'#4

Examples of expressions using powers, roots, and indices

$$(a+b)^2 = a^2 + 2ab + b^2$$

1&`□\*a?/:□<`/:□*"a&`a□*a/:

$$C^k$$

$$V_n^k$$

$$\frac{ax}{y}$$

$$a^{\frac{x}{y}}$$

#b[&j #u[&j0m &`/w7x &`0/w7x

$$x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

&w0:□<:,a□*02a/!:!, "c&`b□7□"a-`;

S

FUNCTION

S

Functions

$$y = f(x)$$

$$f(x) = 2x - 1$$

$$y = 2x - 1$$

&x□<e1w?

&e1w?□<"aw□,"`

&x□<"aw□,"`

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notation using a right-pointing arrow symbol

□90z

$$x \rightarrow y = 2x - 1 \quad \&w□90x□<"aw□,"`$$

$$x \rightarrow 2x - 1 \quad \&w□90"aw□,"`$$

$$f(x) = -2x^2 + 4x + 6 = -2(x - 1)^2 + 8$$

&e1w?□<,"aw/:□*"cw□*"e□<
<,"a1w□,"`?/:□*"g

$$W = (1, 8)$$

#v□<1" `&0□"g?

$$f_{\max} = f(1) = 8$$

The expression on the right gives the coordinates of the vertex of the function $f(x)$.

&e0"w□<e1" `?□<"g

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's inverse function

$$f^{-1}$$

&e{,0

denotation of the inverse function

$$f(x) = \sin x$$

&e1w?□<5rw

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$$f^{-1}(x) = \arcsin x$$

&e{,01w?□<50rw

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$$y = \frac{3}{4}x - 1$$

&x□<"b.w□,"`

$$y^{-1} = \frac{4}{3}x + \frac{4}{3}$$

&x{,0□<"c9w□*"c9

³² The right-pointing arrow is one of the symbols presented in the chapter of the Guide – *Basic geometric symbols*.

³³ The symbols $\max$ "W" and $\min$ "m" are given in the chapter of the Guide – *Operations on sets and set elements*.

³⁴ Trigonometric function symbols are given in the chapter of the Guide – *Trigonometric functions*.

³⁵ As above.

function composed

$$f(g(x))$$

$$u = g(x) = 3x + 2$$

$$f(u) = \sqrt[3]{u}$$

$$f(u) = \sqrt[3]{u}$$

$$f(g(x)) = \frac{\sqrt{3x+2}}{3x+2}$$

$$f(g(x)) = \frac{\sqrt{3x+2}}{3x+2}$$

$$f(g(x)) = \frac{\sqrt{3x+2}}{3x+2}$$

Constans

$$\text{const } j \text{ constans}$$

$$f(x) = \text{const}$$

$$f(x) = \text{const}$$

Signum

$$\text{sgn } r \text{ signum}$$

$$\text{sgn } 5 = 1$$

$$\text{sgn } (-27) = -1$$

$$\text{sgn } 5 = 1 \quad \text{sgn } (-27) = -1$$

Examples of records of the function

$$f_n(x) = nx + 1$$

$$f_n(x) = nx + 1$$

$$f_{n+1}(x) = (n+1)x + 1$$

$$f_{n+1}(x) = (n+1)x + 1$$

$$F_{n_k}(x)$$

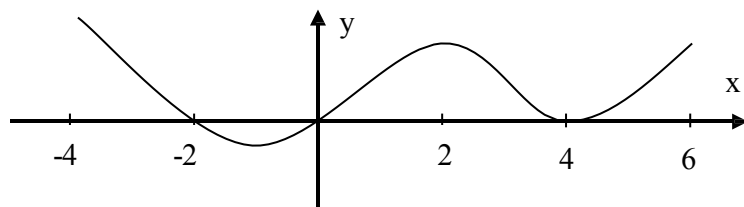
$$F_{n_k}(x) \text{ or } F_{n_k}(x)$$

In the first notation, the character 0 is the beginning of the composite projection, and the empty character ends this projection. In the second notation, the projection ends with the characters 04.

Graphical representation of graphs of polynomials

In lessons devoted to polynomials and quadratic functions, especially when solving inequalities, it is necessary to sketch function graphs with accuracy to the zero points and the sign of the function value.

Example:



Using Braille characters, such a graph sketch can be represented graphically as follows:

eee□,"a□,,,□"i□eee□"c□eee

The numbers in this line are the zero points of the function. The letter "f" in the notation can be treated as "raised" plus signs in Braille, illustrating (and graphically emphasizing) the fact that in certain intervals limited by the zero points, the function takes positive values, and therefore its graph is above the OX axis. The minus signs correspond to the situation where a fragment of the graph runs below the OX axis, i.e., when the function takes negative values. It is therefore easy to see where the graph crosses the OX axis (the function changes sign) and where the graph only "touches" the axis, remaining on the same side (the function does not change sign).

Examples:

A quadratic function with two zero points, with the arms of the parabola pointing downwards:

,,,□"b□eee□"e□,,,

Quadratic function with one (double) zero point, with the arms of the parabola pointing upwards:

eee□,"d□eee

If the notation does not fit on one line, write it on consecutive lines, dividing it at any space.

,,,□,"`□eee□,"`:□,,,□"a9
,,,□"f□eee□"``□,,,

LARGE BRACKET CONNECTING SEVERAL LINES

8□ beginning of expressions enclosed in a large bracket □;7□ new line character

$$\begin{cases} -x & \text{for } x < 0, \\ |x| = \begin{cases} 0 & \text{for } x = 0, \\ x & \text{for } x > 0. \end{cases} \end{cases}$$

!k&w^□<8□,w□ck`□&w□8-"i&0□;7□"i&
 Ck`□&w□<"i&0□;7□w□ck`□&w□n0"i-

or

!k&w^□<8□,w□ck`□&w□8-"i&0
 ;7□"i□Ck`□&w□<"i&0
 ;7□w□ck`□&w□n0"i-

$$\operatorname{sgn} x = \begin{cases} -1 & \text{for } x < 0, \\ 0 & \text{for } x = 0, \\ 1 & \text{for } x > 0. \end{cases}$$

"r&w□<8□,"`□ck`□&w□8-"i&0□;7□"i&
 ck`□&w□<"i&0□;7□"`□ck`□&w□n0"i-

or

"r&w□<8□,"`□ck`□&w□8-"i&0
 ;7□"i□ck`□&w□<"i&0
 ;7□"`□ck`□&w□n0"i-

The newline character is also used to write certain expressions in which the line break occurs locally. An example is the ^{Newton} symbol³⁶. In this case, the newline character is written without spaces.

z;7z newline character with local effect

$$\binom{n}{k}$$

1&m;7j?

Another example of using the new line character is the pattern for combinations with repetitions.

$$-k \quad C_n = \binom{n+k-1}{k}$$

#b9{&j0m□<1m!*j!,"`;7j?

³⁶ Chapter of the Guide – Probability and combinatorics.

EQUATIONS AND SYSTEMS EQUATIONS

Equations

$$2x + 3(x - 2) = x - 10$$

$$2x + 3x - 6 = x - 10$$

$$5x - 6 = x - 10$$

$$5x - x = 6 - 10$$

$$4x = -4 \quad | : 4$$

$$x = -1$$

Due to the continuity of the operations performed, the previously used alternating notation convention has been abandoned:

black print version – Braille version, presenting first the entire solution in black print and then in Braille.

The Braille notation of the solution to the equation in successive lines is clear. However, it takes up a lot of space.

"a-w□*"b1w□,"a?□<w□,"`i
 "a&w□*"bw□,"e□<w□,"`i
 "d&w□,"e□<w□,"`i
 "d&w□,w□<"e□,"`i
 "c&w□<,"c□^□."c
 &w□<,"`

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The same solution to the equation can be written using the logical equivalence sign $\square 0 < 0 \square$, which is ^a very legible ^{separator} for the blind. *The new line character* $\square ; 7 \square$ can also be used as a separator.

"a-w□*"b1w□,"a?□<w□,"`i□0<0
 "a&w□*"bw□,"e□<w□,"`i□0<0
 "d&w□,"e□<w□,"`i□0<0"d&w□,w□<
 <"e□,"`i□0<0"c&w□<,"c□^□."c□0<0
 &w□<,"`

The black-and-white transcription of the above solution seems less legible. $2x + 3(x - 2) =$

$$x - 10 \Leftrightarrow$$

$$2x + 3x - 6 = x - 10 \Leftrightarrow$$

$$5x - 6 = x - 10 \Leftrightarrow 5x - x =$$

$$= 6 - 10 \Leftrightarrow 4x = -4 \quad | : 4 \Leftrightarrow$$

$$x = -1$$

³⁷ Guide Chapter – *Mathematical Logic*.

³⁸ Chapter of the Guide – *A large bracket connecting several lines*.

However, keep in mind:

- Braille is read by touch (with the fingers). For this reason, not everything that looks clear in black print is equally clear to a blind person. Many students definitely prefer the second method of writing.
- In the second method of writing, despite the introduction of additional characters, one line has been saved.
- The use of expression division at any ^{point}³⁹ would certainly result in saving two lines.

systems of equations

When writing systems of equations and subsequent solution steps, you can use large braces and ^{line} breaks⁴⁰

$$\begin{cases} x + 5y = 3 + 3y \\ -4x + y + 2 = 9 - 6x + 2y \\ x + 2y = 3 \\ 2x - y = 7 \end{cases}$$

$$\begin{aligned} &8 \square \&w \square \text{"} dx \square < \text{"} b \square \text{"} bx \square ; 7 \square , \text{"} cw \square \text{"} x \square \text{"} \\ &\text{"} a \square < \text{"} h \square , \text{"} ew \square \text{"} ax \\ &8 \square \&w \square \text{"} ax \square < \text{"} b \square ; 7 \square \text{"} aw \square , x \square < \text{"} f \end{aligned}$$

Descriptive and logical connectives can be used in the notation of successive steps of the solution.

$$\begin{aligned} &\&w \square \text{"} dx \square < \text{"} b \square \text{"} bx \square h \square , \text{"} cw \square \text{"} x \square \text{"} \\ &\text{"} a \square < \text{"} h \square , \text{"} ew \square \text{"} ax \square 0 < 0 \&w \square \text{"} ax \square < \\ &< \text{"} b \square h \square \text{"} aw \square , x \square < \text{"} f \end{aligned}$$

The black print notation of the subsequent steps using descriptive conjunctions is as follows:

$$\begin{aligned} &x + 5y = 3 + 3y \text{ i } -4x + y + \\ &+ 2 = 9 - 6x + 2y \iff x + 2y = \\ &= 3 \text{ and } 2x - y = 7 \end{aligned}$$

The clarity of the subsequent steps of the solution for sighted people will certainly be debatable. However, many blind students choose a linear notation.

³⁹ Chapter of the Guide – *Transferring parts of mathematical expressions to the next line.*

⁴⁰ Guide Chapter – *Large bracket connecting several lines.*

Determinant symbols and matrix s

Dashes for the determinant

$$\begin{vmatrix} \cdots \\ \cdots \end{vmatrix} \quad \begin{matrix} !kz---z^{\wedge} \\ kz---z^{\wedge} \end{matrix}$$

Determinant module

$$\begin{vmatrix} \cdots \\ \cdots \end{vmatrix} \quad \begin{matrix} !kkz---z^{\wedge\wedge} \\ kkz---z^{\wedge\wedge} \end{matrix}$$

Parentheses for matrices

$$\begin{pmatrix} \cdots \\ \cdots \end{pmatrix} \quad \begin{bmatrix} \cdots \\ \cdots \end{bmatrix} \quad \begin{matrix} 1z---z? \\ 1z---z? \end{matrix}$$

Determinant

$$\det 6cds$$

For the system of equations

$$\begin{cases} x + 2y = 3 \\ 2x - y = 7 \end{cases}$$

determinant:

$$W = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = 1 \cdot (-1) - 2 \cdot 2 = -5$$

$$\begin{matrix} \#v \square < !k^{\wedge} \square \square "a^{\wedge} \\ k^{\wedge} "a \square, ^{\wedge} \square < ^{\wedge} -1, ^{\wedge} ? \square, "a- \\ -"a \square < , "d \text{ or} \end{matrix}$$

$$6cds 1^{\wedge} \square "a; 7^{\wedge} "a \square, ^{\wedge} ? \square < ^{\wedge} -1, ^{\wedge} ? \square, \\ , "a - "a \square < , "d$$

$$|W| = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = |1 \cdot (-1) - 2 \cdot 2| = |-5| = 5$$

$$\begin{matrix} !k \#v^{\wedge} \square < !kk^{\wedge} \square \square "a^{\wedge\wedge} \\ kk^{\wedge} "a \square, ^{\wedge\wedge} \square < !k^{\wedge} -1, ^{\wedge} ? \square, \\ , "a - "a^{\wedge} \square < !k, "d^{\wedge} \square < "d \text{ or} \end{matrix}$$

$$\begin{matrix} !k 6cds 1^{\wedge} \square "a; 7^{\wedge} "a \square, ^{\wedge} ?^{\wedge} \square < !k^{\wedge} -1! \\ , ^{\wedge} ? \square, "a - "a^{\wedge} \square < !k, "d^{\wedge} \square < "d \end{matrix}$$

representation in matrix form:

$$\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

$$\begin{matrix} 1^{\wedge} \square \square "a? \\ 1^{\wedge} "a \square, ^{\wedge} ? \end{matrix}$$

LOGARITHMS

function notation

log 5^k logarithm with base 10ln 5^k natural logarithm

$z^{\{z}$ preceding sign
base of the logarithm other than e or 10⁴¹

The base of the logarithm is treated as a left-hand superscript and is given before the logarithm sign.

$$\log 1000 = 3$$

$$5^k \text{ " " i i i } \square < \text{ " b}$$

$$\ln e^2 = 2$$

$$5^k \square \& d / : \square < \text{ " a}$$

$$\frac{\ln(x+2)}{x-2}$$

$$: 5^k 1 \& w \square * \text{ " a ? } \square 7 \square w \square, \text{ " a ; or}$$

$$5^k 1 \& w \square * \text{ " a ? } 7 w !, \text{ " a}$$

$$\ln \frac{x+2}{x-2}$$

$$5^k \square \& w ! * \text{ " a } 7 w !, \text{ " a or}$$

$$5^k : \& w \square * \text{ " a } \square 7 \square w \square, \text{ " a ;}$$

$$\log_a(x_1 x_2) = \log_a x_1 + \log_a x_2$$

$$\{ \& \text{ ` } 5^k 1 w 0 0 - w 0 : ? \square < \{ \& \text{ ` } 5^k w 0 0 \square * \\ * \{ \& \text{ ` } 5^k w 0 :$$

$$\log_a x^m = m \cdot \log_a x$$

$$\{ \& \text{ ` } 5^k w / l \square < l - \{ \& \text{ ` } 5^k w$$

$$\log_x y = \frac{1}{\log_y x}$$

$$\{ \& w 5^k x \square < \text{ " ` } 7 \{ x 5^k w$$

⁴¹ Chapter of the Guide – Left-sided indicators.

GEOMETRY

Basic geometric symbols

⊥	"_	perpendicular
⊈	□)"_	non-perpendicular
∥	!kk	parallel
⊈	□)!kk	non-parallel
~	□>	is similar
⋈	□)>	is not similar
≡	□<<	is compatible
≢	□)<<	is not compatible
∠	68	angle
⊓	6'	right angle
△	6.	triangle
⊓	6..	right triangle
□	6<	square
□	6\$	rectangle
○	6(	circle
⊙	6)	diameter
'	z)	prim
"	z))	bis
	#9z	section
	#99z	straight
⌒	#1	upper arc
⌒	;1	lower arch
→	90	arrow pointing to the right
←	09z	arrow pointing left
↔	090z	arrow pointing right and left

$$AB \parallel CD$$

$$AA' \nparallel BB'$$

$$\#`a\Box!kk\#bc\#``)\Box)"-\#aa)$$

$$\sphericalangle B = \sphericalangle ABC$$

$$a \nparallel b$$

$$68\#a\Box<68\#`ab -`)\Box)!kk-a$$

$$\triangle ABC \sim \triangle A_1 B_1 C_1 \sim \triangle A' B' C'$$

$$6.\#`ab\Box>6.\#`00a00b00\Box>6.\#`)a)b)$$

$$\widehat{ACB}$$

$$C \in \overline{AB}$$

$$\overline{AC} \perp \overline{DB}$$

$$\begin{array}{l} \#1\#`ba \quad \#b\Box!d\#9\#`a \quad \#9\#`b\Box"- \#9\#ca \\ \text{or } \#b\Box!d\#9`a \quad \#9\#`b\Box"- \#9ca \end{array}$$

's vector notation

The following key characters are used to denote vectors:

$$\rightarrow \quad \#90z \quad \text{vector pointing to the right}$$

$$\leftarrow \quad \#09z \quad \text{vector pointing left}$$

The vector arrow key is a parenthesizing character⁴². This means that it encompasses the entire expression that follows it.

$$\begin{array}{c} \rightarrow \quad \rightarrow \\ AB = -DC \end{array}$$

$$\#90\#`a\Box<,\#90\#CB$$

$$\begin{array}{c} \rightarrow \quad \rightarrow \quad \rightarrow \\ V = v_1 + v_2 \end{array}$$

$$\#90\#u\Box<\#90\&u00\Box*\#90u0:$$

$$\text{straight line } \overrightarrow{PP_0} \parallel u$$

$$\#99\#000(\Box!kk\#90\&t$$

$$\alpha = \sphericalangle^{\rightarrow\rightarrow}(v, u)$$

$$;\`)\Box<681\#90\&u0\Box\#90t?$$

$$\begin{array}{c} \rightarrow \quad \rightarrow \\ \text{Vectors } AB \text{ and } DC \text{ are opposite.} \end{array}$$

$$\begin{array}{c} \text{A line passing through the points} \\ P \text{ and } P_0 \text{ parallel to vector } u. \end{array}$$

⁴² Chapter of the Guide – Brackets written above or below an expression.

al geometry

Distance of a point from a line

$$l: Ax + By + C = 0, P = (x_p, y_p)$$

$$d = \frac{|Ax_p + By_p + C|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{|Ax_p + By_p + C|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{|Ax_p + By_p + C|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{|Ax_p + By_p + C|}{\sqrt{A^2 + B^2}}$$

The area of a triangle with vertices

$$A = (x_a, y_a),$$

$$B = (x_b, y_b),$$

$$C = (x_c, y_c).$$

$$S_{\triangle ABC} = \frac{1}{2} \begin{vmatrix} x_a & y_a & 1 \\ x_b & y_b & 1 \\ x_c & y_c & 1 \end{vmatrix}$$

$$S_{\triangle ABC} = \frac{1}{2} \begin{vmatrix} x_b - x_a & y_b - y_a \\ x_c - x_a & y_c - y_a \end{vmatrix}$$

$$S_{\triangle ABC} = \frac{1}{2} \begin{vmatrix} x_b - x_a & y_b - y_a \\ x_c - x_a & y_c - y_a \end{vmatrix}$$

⁴³ Relevant symbols in the Guide chapter – *Determinant and matrix symbols*.

TRIGONOMETRY

's angular measure

full notation	abbreviated notation	
$^{\circ}$	=	(angle degree symbol
$'$	$\text{=})$	) angle minute sign ⁴⁴
$''$	=))	)) angle second sign ⁴⁵

$$\alpha = 30^{\circ}$$

$\beta = \frac{1}{50}$

$$\beta = \frac{1}{50}$$

$\beta = \frac{1}{50}$

The full degree notation is used in the above expression because if the abbreviated notation abbreviation $\beta = \frac{1}{50}$, the expression would be read as $\beta = \frac{1}{50}$

$$19^{\circ} 23' 47''$$

$19^{\circ} 23' 47''$

The notation of minutes and seconds of angle almost always requires the use of the full notation.

Arc measure angle

radian $\frac{\pi}{180}$ or $\frac{1}{180}$

$$90^{\circ} = \frac{\pi}{2} \text{ rad}$$

$90^{\circ} = \frac{\pi}{2} \text{ rad}$

or

$90^{\circ} = \frac{\pi}{2} \text{ rad}$

$$1 \text{ rad} = \frac{180^{\circ}}{\pi} \approx \frac{180^{\circ}}{3.14159} \approx 57^{\circ} 17' 45''$$

$1 \text{ rad} = \frac{180^{\circ}}{\pi} \approx \frac{180^{\circ}}{3.14159} \approx 57^{\circ} 17' 45''$

⁴⁴ Due to ambiguities, the abbreviated notation for angular minutes is used very rarely.

⁴⁵ Due to ambiguities, the abbreviated notation for angular seconds is used very rarely.

nd trigonometric functions

Trigonometric function symbols

sin 5r sine

cos 5b cosine

tg 5s tangent

ctg 57 cotangent

sec 5, secant

cosecant 51 cosecant

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

Examples of trigonometric functions

$$\frac{\cos \alpha}{2}$$

$$\cos \left(\frac{\alpha}{2} \right)$$

$$\tan \left(\frac{90^\circ - \alpha}{2} \right)$$

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \frac{\alpha}{2}$$

$$\tan \frac{90^\circ - \alpha}{2}$$

Using a space after the function name before the algebraic fraction makes the fraction is the argument of the function.

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha}$$

$$\sin \alpha = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\sin \alpha = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos\left(-\frac{\pi}{3}\right) = \cos\left(-\frac{\pi}{3} \cdot 180^\circ\right) = \cos(-60^\circ) = \frac{1}{2}$$

5b1,;o79?□<5b1,::;o79□7□;o;-
-""gi(?□<5b1,"ei(?□<"" :

$$\csc 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$$

51"bi(□<""75r"bi(□<""7"" :□<"a

Inverse trigonometric functions

Notations for inverse trigonometric functions

arcsin 50r arcus sinus

arccos 50b arcus cosine

arctg 50s arcus tangens

arcctg 507 arcus cotangent

arcsec 50, arcus secant

arccosec 501 arcus cosecans

$$f(x) = \tan x$$

&e1w?□<5sw

$$f^{-1}(x) = \arctg x$$

&e{,041w?□<50sw

$$\arcsin(30^\circ + n \cdot 360^\circ) = \frac{1}{2}$$

for $n = 0, \pm 1, \pm 2, \dots$

50r1"bi(□*&m-"bei(?□<"" :

ck`□&m□<"i&0□*, ""&0□*, "a&0□---

MATHEMATICAL LOGIC

logical symbols

$\vee$	$\square 20$	or
$\wedge$	$\square / 0$	and
$\Rightarrow$	$\square < 0$	is
$\Leftrightarrow$	$\square 0 < 0$	if and only if
$\sim$	$\square 9)$	no (it is not true that)

Quantifiers

$\vee$ or $\exists$ $\%>$ specific (there exists such that) $\wedge$

or $\forall$ $\%0$ general (for every ...)

$a \vee b$

$\& \square 20 a$

$((p \wedge q) \vee (q \wedge r)) \Rightarrow v$

$11 \& o \square / 0 p ? \square 20 1 p \square / 0 \text{œ} ?? \square < 0 u$

$\forall x \ x^2 \geq 0$

$\% 0 \& w \square w / : \square n < " i$

$\exists x \ x + 1 < 0$

$\%> \& w \square w \square * " \square 8 - " i$

$\sim \forall x \ p(x) \Leftrightarrow \exists x \sim p(x)$

$9) \% 0 \& w \square o 1 w ? \square 0 < 0 \%> w \square 9) o 1 w ?$

PROBABILITY CALCULATION AND COMBINATORICS

Factorial

! 5 □ factorial

$$5! = 12345$$

"d5 □ < " - "a - "b - "c - "d

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

After the factorial sign, a space should be left, and if for some reason we need to fill it, we use the ! sign.

1 & m; 7 j ? □ < : m 5 □ 7 □ j 5 ! 1 m □ , j ? 5 ! ;

$$C_n^k = \binom{n}{k}$$

#b { & j 0 m □ < 1 m ; 7 j ?

$$e_n^k = \binom{n+k-1}{n-1} = \binom{n+k-1}{k}$$

#b 9 { & j 0 m □ < 1 m ! * j ! , " ` ; 7 m ! , " ` ? □ < < 1 m ! * j ! , " ` ; 7 j ?

$$V_n^k = \frac{n!}{(n-k)!}$$

#u { & j 0 m □ < : m 5 □ 7 □ 1 m □ , j ? 5 ! ;

$$\bar{V}_n^k = nk$$

#u 9 { & j 0 m □ < m / j

LIMITS

Symbols used in the notation limits

$\lim$ "k
 ∞ "\$ symbol of infinity
 $\rightarrow$ "90 "strives for"

The symbol "tends towards" is a combination of the filler character

! and the abbreviated arrow symbol 90.

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

"k0&w!90"\$□"7w□<"i

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$$

An empty character was used before expression ¹
x

□. It closes the simple projector.

"k0&w!90"i=*□"7w□<"\$

$$\lim_{n \rightarrow \infty} \frac{n+1}{n} = 1$$

"k0&m!90"\$□m!*□"7m□<"`

$$\lim_{x \rightarrow \infty} 2^{-x} = 0$$

"k0&w!90"\$□"a/,w□<"i

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

"k0^c&w!90"i□:e1w□*^c&w?□,e1w?□7
7□^c&w;

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} (x + 2) = 4$$

"k0&w!90"a□:w/:□,"c□7□w□,"a;□<
<"k0&w!90"a□1w□*"a?□<"c

$$\lim_{e \rightarrow x_0} f(x)$$

&d#/"k00w!90w0(□e1w?#4

The # symbol is the start symbol for the detailed projector, the 0 starts a compound projection. A compound projector ends with an empty character, and a detailed one with #4.

DERIVATIVE

S

derivative

$$f'(x) = \frac{dy}{dx} = y'$$

$$f(x) = \frac{1}{x^2} = x^{-2}$$

∂ partial derivative sign

$$g(x,y) = \frac{\partial f(x,y)}{\partial x}$$

$$f(x,y) = x^2 + y^2$$

INTEGRALS

$\int$ integral sign
 $\int$ integral line

$$\int f'(x) dx = f(x) + C$$

$$\int_1^2 x^2 dx = \frac{1}{3} x^3 \Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

$$\int_{x_1}^{x_2} f(x) dx = F(x_2) - F(x_1)$$

definite integral

$$\int_0^2 3x^2 dx = x^3 \Big|_0^2 = 2^3 - 0^3 = 8$$

$$\int_0^2 3x^2 dx = x^3 \Big|_0^2 = 2^3 - 0^3 = 8$$

$$\int_0^2 3x^2 dx = x^3 \Big|_0^2 = 2^3 - 0^3 = 8$$

The symbols used in Braille notation cited and discussed in the Guide, as well as examples of various expressions, indicate the importance of precise and consistent use of Braille characters.

- The same Braille symbol can have multiple meanings.
- The position of a character in a mathematical text plays an important role.
- A break in the notation (space), referred to as a "blank character," is particularly important.

Due to the proximity of the "blank character" in Braille mathematical notation, there are 6 groups of characters⁴⁶. Examples of characters from each group are given below:

Group A – characters written on the left side with or without a space.

$\sqrt{}$	$\overset{\text{capital letter}}{\text{Latin letter}}$	tg
$2z$	$\#z$	$5s$

Group B – characters written with a space on the left

$+$	$\geq$	$=$
$\square^*z$	$\square n < z$	$\square < z$

Group C – characters written without spacing on the left

$\overset{\text{indicator}}{\text{right-hand}} \text{upper}$	$'$ (prim)	end of projector
$z\{z$	$z)$	$z4$

Group A' – characters written on the right side with or without

spacing const	rad	$\triangle$
$"j$	$"Q$	$6.$

Group B' – characters written with a space on the right

$!$	$\{$ (large)
$5\square$	$8\square$

Group C' – characters written without spacing on the right

$\overset{\text{print character}}{\text{bold}}$	$>$
$z\#z$	$n0z$

⁴⁶ Chapter of the Guide – *Sequence of characters*.

The immediate proximity of certain groups of characters prevents the above conditions for correct notation from being met. An example of this are characters from group C occurring immediately after characters from group B*. One command requires a space between two characters, while another excludes "empty characters." The definitions of symbols adopted in mathematical notation minimize the number of such cases. However, it is difficult to exclude them completely. In order to resolve such situations, different levels of authority have been assigned to individual definitions.

- The definitions in groups C and C' are absolutely binding.
- Next in importance are the definitions in groups B and B'.
- Groups A and A' are last.

In conflict situations, the following rules regarding the sequence of mathematical symbols should be applied:

1. Two consecutive symbols are written **directly after each other** in two cases:

- if the first is from group C' and the second is any other, or
- The first is arbitrary, and the second is from group C.

The opening brace
(group B')

8□

> (group C')

n0z

Upper right angle bracket
right-hand superscript (group C)

z{z

= (group B)

□<z

Total

8{z

n0<z

2. There must be a "blank character" between two consecutive characters when

- the first is from group B' and the second is from B or A, or
- the second is from group B and the first is from B' or A'.

! (group B')

5□

+ (group B)

□*with

Total

5□*z

3. If the first character is from group A' and the second is from A or vice versa, you can leave a space between them or not, depending on what is easier to read.

– COMMENTS

Physical sciences use mathematical tools to present their content and analyses. Therefore, in most cases, physical notation is essentially no different from mathematical notation. However, physical relationships use slightly different symbols, and physical quantities are often accompanied by complex indices. Physical units and their conversions also play an important role. The following chapters will discuss issues related to physical notation in greater detail. However, this content is an integral part of mathematical notation. It is assumed that the reader is familiar with the chapters of the Guide devoted to mathematical notation.

INDICES AND "SYMBOLS" IN PHYSICAL FORMULAS⁴⁷

The use of indices and "symbols" in physical quantity symbols is very widespread. In many cases, they are an integral part of the symbol, while in others they allow the state or course of a process to be precisely determined. They emphasize the interrelationships between quantities and often indicate certain correlations, opposites, and differences. The use of indices and "marks" in notation certainly makes it more orderly, transparent, and thus more understandable. For these reasons, this section of the Guide will present and discuss selected relationships from various branches of physics in which additional markings in the form of indices and "marks" are used.

In most cases, right-hand indices are used in physical quantity symbols.⁴⁸

z_0z - lower right-hand index

$z\{z$ - upper right-hand index

$$v = \sqrt{v_1^2 + v_2^2}$$

$\&u\Box<02u00/;!*\u0:/:$

Indices specify the designation of a physical quantity and emphasize the relationships between quantities.

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

$$C_0 = \frac{Q}{V_0}$$

$;\mathbf{k}\Box<5'\mathbf{k}''\mathbf{a}7\#\mathbf{s}0''\text{`}:$

$\#\mathbf{b}0(\Box<\mathbf{P}7\mathbf{u}0($

or $\#\mathbf{b}0(\Box<\#\mathbf{P}7\#\mathbf{u}0($

$$U_{sk} = \frac{U_0}{\sqrt{2}}$$

$$x = x_0 - v t$$

$\#\mathbf{T}0\&\mathbf{rj}\Box<\#\mathbf{t}0(72''\mathbf{a}$

$\&\mathbf{w}\Box<\mathbf{w}0(\Box,\mathbf{us}$

$$\begin{matrix} s_1 = \left(\begin{matrix} t_1 \\ t \\ 2 \end{matrix} \right)^2 \\ s_2 \end{matrix}$$

$\&\mathbf{r}007\mathbf{r}0:\Box<1\mathbf{s}007\mathbf{s}0:?:/:$

⁴⁷ Definitions of right-hand, left-hand, and "mark" indicators, detailed spelling rules, and relevant tables are provided in the Guide chapter – *Powers and Indicators*.

⁴⁸ Chapter of the Guide – *Key symbols for exponents and right-hand indices*.

$$I_1 = \frac{I_0 R_1}{R_1 + R_2}$$

#h00□<h0(Q007OE00!*Q0:

or #h00□<#h0(#Q007#OE00!*#Q0:

or #h00□<:#h0(#Q00□7□#OE00!*#Q0::

$$\eta = \frac{T_1 - T_2}{T_1}$$

;4□<#s00!,S0:7s00

or ;4□<#s00!,#S0:7#s00

or ;4□<:#s00□,#S0:□7□#s00;

$$W_{AB} = -GMm \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

#v0`a□<,fl&l1""`7OE0#`□,"`7&OE0!
!#a?

The first line of the above formula in Braille is broken after the subscript "B".

The mathematical expression was interrupted at a place where there is no empty character, so the ! character is used as a carry character. This character must be repeated at the beginning of the next line.⁴⁹

By using capital letters redundantly, you can also write:

#v0#`a□<,#f#l&l1""`7OE0#`□,"`7
7&OE0#a?

The line was broken at the fraction mark.

In some physical quantity notation, so-called "symbols" are used.⁵⁰ Below is a list of the most commonly used "symbols" in physical notation.

'	z)	prim	°	z(	circle
"	z))	bis	.	from:	period
+	z*	plus	^	z/	roof
-	wit h,	minus	~	z>	falka
—	z9	line	*	z.	asterisk

Symbols are written after the symbol to which they refer. They are assigned the same row as the main symbol. The position of the symbol relative to the main symbol is determined by the corresponding key character between it and the symbol.

⁴⁹ Chapter of the Guide – *Moving parts of mathematical expressions to the next line.*

⁵⁰ Chapter of the Guide – *So-called "tags."*

The key characters used for symbols are as follows:

$\mathbf{Z}=\mathbf{Z}$ upper mark

$\mathbf{Z};\mathbf{Z}$ lower stamp

The lower stamp key character is always valid. The upper stamp key character can be omitted if the notation is unambiguous.

$$E = \frac{1}{2}mv^2$$

Average values are often indicated by a line above the corresponding symbol.

$\#d9\Box<"\` :&lu9/:$

$\tilde{v}$

π

$;\mathbf{m}>$

$;\mathbf{o}.$

Arrows in physical records

Regular arrows⁵¹

Arrows can be used in descriptions to indicate the direction of changes, as one of the expressions of an indicator, etc.

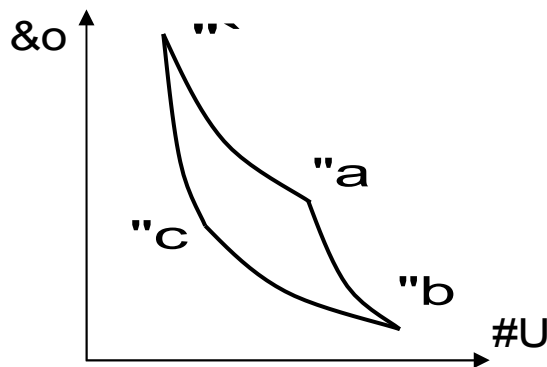
The key symbols for such arrows are as follows:

$\rightarrow$ $\Box90$ arrow pointing to the right

$\leftarrow$ $\Box09$ arrow pointing left

$\leftrightarrow$ $\Box090$ arrow pointing right and left The use of the regular

arrow symbol is shown below in the description of the Carnot cycle.



1 $\rightarrow$ 2 isothermal expansion

" $\Box90$ " $\Box$ hynsd $\Box$ Elhbymd $\Box$ CEnyoCE4%` , mhd

2 $\rightarrow$ 3 adiabatic expansion

" $\Box90$ " $\Box$ b $\Box$ `ch` $\Box$ a`sxbymd $\Box$ CEnyoCE4%` , mhd

⁵¹ Chapter of the Guide - Basic geometric symbols.

In processes $3 \rightarrow 4 \rightarrow 1$, the gas releases heat Q_2

#V□oCEnbdr`bg□"b□90"c□90" `□f`y
ncc`id□bhdo1n□#p0:

The ordinary arrow symbol can also be used as one of the words in an indicator symbol. In the case of indicators consisting of several words, after the last word we write 4 the end of projection symbol or insert a blank character.

$$W_{2 \rightarrow 3} = - W_{4 \rightarrow 1}$$

#v0"a!90"b□<,#v0"c!90" `

The ! character in the index is a placeholder. Its use in this case is necessary. The above expression written incorrectly without this character:

#v0"a□90"b□<,#v0"c□90" `

would be unambiguously read as:

$$W_2 \rightarrow 3 = - W_4 \rightarrow 1$$

It would also be a significant error to omit the space in the definition of the symbol for a regular arrow.

#v0"a90"b□<,#v0"c90" `

The first seven characters are W_2 , followed by 3. After the equal sign, we have W_4 ,

and the subsequent characters are 1.

Arrows in vector magnitude notation

The following key characters are used to denote vector quantities⁵²

→ #90z vector turned to the right

← #09z vector pointing left

$$\vec{v} \quad \vec{F} = m \vec{a}$$

#90&u #90#e□<&l#90`

The key symbol of the vector arrow written before the quantity symbol is a parenthesis.⁵³ This means that it encompasses the entire expression that follows it.

$$\Delta p \quad F_{AB} = - F_{BA}$$

#90^c&O #90#e0#`a□<,#90#e0#A@

⁵² Guide Chapter – Vector notation.

⁵³ Guide Chapter – Brackets written above or below an expression.

$\rightarrow \quad \rightarrow$

$$F_{1,2} = -F_{2,1}$$

 $\#90\#e0'' \&0!"a\Box<,\#90e0"a\&0!"'$
 $\rightarrow \quad \rightarrow \quad \rightarrow$

$$\Delta r = r_2 - r_1$$

 $\#90^c\&\text{OE}\Box<\#90\text{OE}0:\Box,\#90\text{OE}00$
 $\rightarrow$

$$|r_2| = x_2$$

 $!k\#90\&\text{OE}0:^{\wedge}\Box<w0:$
 $\rightarrow \quad \rightarrow$

$$a = \frac{\Delta v}{\Delta t}$$

 $\#90\&' \Box<\#90^c\&u7^c\&s$
 $\rightarrow \quad \rightarrow$

$$\Delta p_1 = m_1 \Delta v$$

 $\#90^c\&o00\Box<|00\#90^c\&u$
 $\rightarrow$

$$\vec{E} = \frac{\vec{F}}{q_0}$$

 $\#90\#d\Box<\#90\#e7\&p0($
 $\rightarrow \quad \rightarrow \quad \rightarrow$

$$M = F \times r$$

 $\#90\#L\Box<\#90\#e\Box'\#90\&\text{OE}$

$$W = \int_{F_s} \rightarrow \rightarrow$$

 $\#v\Box<\#90\#e\Box-\#90\&r$

Arrows under an expression

Arrows used as symbols written above or below an expression. ⁵⁴

$\rightarrow \quad \Box;90z$ right arrow written below the expression

$\leftarrow \quad \Box;09z$ left arrow written below the expression

$$N_0 \rightarrow \frac{T_{1/2}}{2} \quad \frac{N_0}{2} \rightarrow \frac{T_{1/2}}{4} \quad \frac{N_0}{4} \xrightarrow{\frac{T_{1/2}}{8}} \frac{N_0}{8}$$

In this expression, the 6th point before T is the beginning of the expression written above the arrow, and the 3rd point is the end of that expression.

 $\#M0(\Box;90\&\#s0'' :-\Box m0(7:\Box;90\&\#s0!$
 $!'' :-\Box m0(7.\Box;90\&\#s0'' :-\Box m0(7'$

The multiplication sign (cross) is used to denote the vector product of two vectors.

The dot product of two vectors is written using the multiplication sign – a dot.

Arrows precede the expression that appears above or below the arrow.

⁵⁴ Arrow symbols also used in chemical formulas; section of the Guide – *Arrow symbols in chemical notation*.

The expression can also be written using the capital letter "excessively":

#MO(□;90&#s0"⠨:-□#m0(7:□;90&#s!
!O"⠨:-□#m0(7.□;90&#s0"⠨:-□#m0(7.⠨

The first line of the expression has been broken at a point that is not an "empty character". In this case, on

At the end of the line, place a fourth dot ⁵⁵⠨. This character is repeated at the beginning of a new line.

Left-hand indices in physical expressions

In some physical expressions, it is necessary to use left-hand indices.⁵⁶

abbreviated notation

full notation

□{z " {z - upper left-hand index

□Oz "Oz - lower left-hand index

In general, the abbreviated notation is sufficient. The full notation of left-hand indices should be used in cases where it is necessary to avoid ambiguity.

²¹⁵₈₄ After $\alpha \rightarrow$ ²¹¹₈₂ Pb + ⁴₂ He + 7.4MeV

{:0>O' .#On□;90;`□{:000':#Oa□*
{.0:#Gd□"f0c6#ld#U

The above reaction is written using the chemical element symbol. The chemical element symbol ⁵⁷ is a whole and should not be treated as an algebraic expression. The second letter in two-letter element symbols is not preceded by a lowercase Latin letter.

The word symbol for the unit is preceded by the name symbol ⁵⁸. Other rules for the use of alphabetical characters also apply to the notation of units. Within the symbol, each capital letter refers only to the letter preceding it.⁵⁹

¹⁴₆ C $\rightarrow$ ¹⁴₇ N + ⁰₋₁ β + ⁰₀ γ

{0.0*#b□90{0.0<#m□*{(0,0;a□*
*{(0(;m>

¹₀ n $\rightarrow$ ¹₁ p + ⁰₋₁ β + ⁰₀ γ

{00(&m□;90;a,□{000&o□*{(0,0!
!;a□*{(0(;m>

The first line of the expression was broken before the β particle symbol, which begins the second line.

The break occurred where there is no empty grid. The transfer symbol is ⁶⁰⠨. This symbol

starts the next line.

⁵⁵ Guide Chapter – Moving parts of mathematical expressions to the next line.

⁵⁶ Guide Chapter – Left-hand side indicators.

⁵⁷ Guide Chapter – Root symbols.

⁵⁸ Guide Chapter – Units and operations on units.

⁵⁹ Guide Chapter – Verbal symbols, sign of change (rule 2).

⁶⁰ Guide chapter – Transferring parts of mathematical expressions to the next line.

EXAMPLES OF WRITING VARIOUS PHYSICAL FORMULAS IN BRAILLE

Selected issues of Braille transcription

Braille transcription allows for some flexibility in notation. However, whenever possible, the shortest notation should be used. Below are examples of different correct notations for the same printed expressions. The boxes show the titles of the chapters of the Guide, which discuss in detail the use of appropriate abbreviations.

v_1	v_1
$\&u0^{\text{`}}$	$\&u00$
v_0^2	v_0^2
$\&u0^{\text{`}}i/^{\text{`}}a$	$\&u0(/:$
$\frac{R}{2}$	$\frac{R}{2}$
$\#Q7^{\text{`}}a$	$\#Q7:$
$\frac{mv^2}{2}$	$\frac{mv^2}{2}$
$\&lu=9/:7:$	$\&lu9/:7:$
$\frac{T_1-T_2}{T_1}$	
$:\#S00\Box,S0:\Box7\Box s00;$	or
$:\#S00\Box,\#S0:\Box7\Box\#s00;$	
$\frac{T_1-T_2}{T_1}$	
$\#S00!,S0:7s00$	
$\#S00!,\#S0:7\#s00$	

Chapter – Key characters for exponents and right-hand indices (rule 1).

Chapter – Simplified notation of fractions (rule 2).

Chapter – So-called "stamps."

Chapter – Simplified notation of fractions (rule 4).

Some formulas can be written using simple root symbols.

$v_k = \sqrt{2gh}$
$\&u0j\Box<2^{\text{`}}a\&fg$

Chapter – Simple root sign.

However, a similar pattern requires the use of the complex projector sign 0.

$$t = \sqrt{\frac{2h}{g}}$$

&s□<02"a&g7f

Section – Complex root sign.

Complex formulas require a more detailed explanation.

$$t = \frac{\sqrt{v_0^2 + 2gh} - v_0}{g}$$

The above expression in standard notation using fraction start and end symbols will have the following form.⁶¹

&s□<:02u0(/:!"a&fg□,u0(□7□f;

In standard notation, the numerator of a fraction is enclosed between the fraction mark and the fraction bar. The operation symbols between the individual terms of the numerator and denominator are preceded by "blank characters." The root symbol is preceded by the composite projection symbol 0. The operation of the composite root symbol is interrupted by a blank character.

The abbreviated notation of the above expression will have the following form:

&s□<:02u0(/:!"a&fg□,u0(□7f

In the abbreviated notation, the fraction bar is in the zero row, and the numerator and denominator of the fraction are in the first row. The root sign precedes the compound projection sign 0. The root operation ends with a space. There is no space after the fraction bar, and after the short denominator there is a fraction end sign. In this example, the numerator is too long to omit the fraction start sign.

$$h_{max} = \frac{v_0^2 \sin^2 \alpha}{2g}$$

&g0"w□<u0"ix/:7"a&f□<:"au0(!
!5r/::`7"a&f

In the above Braille notation:

- the abbreviation of the word symbol *max* - "W" was used.⁶²
- The fraction bar ends the simple power exponent projector.⁶³

⁶¹ Guide Chapter – Full fraction notation.

⁶² Guide Chapter – Other symbols used in set algebra.

⁶³ Guide Chapter – Key signs for exponents and right-hand indices, rule 2.

The use of the multiplication sign between two fractions is explained by the following example:

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_0}{r^2}$$

full notation

#e<:"` 7 "c;o;d0(; -: &pp0(7 7 0E/::;

abbreviated notation

#e<:"` 7 "c;o;d0(-&pp0(7 0E/:

In the case of abbreviated notation of fractional products, the multiplication sign with a space must be used

□-. The "blank character" signals the end of the fraction. In the notation, the lowercase Greek letter before ε is used redundantly.

Examples of expressions using complex characters keys

Below are examples of patterns that require the use of complex projections in Braille transcription. They begin with the sign 0. The signs 04 end this projection.

$$r = \sqrt{\frac{10N}{\pi N_0}}$$

&OE<02""` i#m7 ; o#M0(

$$T_2 = 2\pi \sqrt{\frac{l}{g+a}}$$

#s0: <"a;o02&k7f!*`

$$U_0 = U_{R_1} + U_{R_2}$$

#T0(<T00Q00□*T00Q0:

or

#T0(<#T00#Q00□*#T00#Q0:

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

&l<:l0(7 02""` !, 1u7b?/::;

$$N = N_0 e^{-\frac{\ln 2}{T_{1/2}}}$$

#m<#m0(&d0/, 5'k"a7#s0""` :04

UNITS AND OPERATIONS ON UNITS

Verbal symbols, e sign of denomination

In unit notation, a key symbol is used for verbal symbols, in the case of units called the name symbol.

6z - key symbol for word symbols, name symbol

Basic units of the SI system

Quantity	Unit name	Symbol	Braille notation
Length	meter	m	6l
time	second	s	6r
mass	kilogram	kg	6jf
current	ampere	A	6#@
temperature	kelvin	K	6#J
luminosity	candela	cd	6bc
amount of matter	mol	mol	6lnk

Supplementary units of the International System of Units (SI)

Quantity	Unit name	Symbol	Braille notation
plane angle	radian	rad	6OE`c or "Q
solid angle	steradian	sr	"OE/:

Derived units of the SI system

Quantity	Unit name	Symbol	Braille notation
Surface area	square meter	m ²	6l/:
volume	cubic meter	m ³	6l/9
speed	meter per second	m/s	6l7r
acceleration	meter per second squared	m/s ²	6l7r/:
force	newton	N	6#m
pressure	pascal	Pa	6#o`

Quantity	Unit name	Symbol	Braille notation
work, energy	joule	J	6#i
power	watt	W	6#v
frequency	hertz	Hz	6#gy
density meter	kilogram per cubic meter	kg/m ³	6jf7l/9
electric charge	coulomb	C	6#b
field strength electric	volt per meter	V/m	6#u7l
voltage	volt	V	6#u
electrical capacitance	farad	F	6#e
electrical resistance	ohm	Ω	6^v
conductivity electrical	siemens	S	6#r
magnetic flux	weber	Wb	6#va
magnetic induction	tesla	T	6#s
inductance	henry	H	6#g
luminous flux	lumen	lm	6kl
illuminance	lux	lx	6kw
body activity rel of radioactivity	becquerel	Bq	6#ap
focusing power	diopter	D	6#c

SI unit prefixes

Prefix	Multiplier	Abbreviation	Braille notation
atto	10^{-18}	a	6
femto	10^{-15}	f	6e
pico	10^{-12}	p	6o
nano	10^{-9}	n	6m
micro	10^{-6}	μ	6;l
milli	10^{-3}	m	6l
cents	10^{-2}	c	6b
deci	10^{-1}	d	6c
deka	10^1	da	6c`
hecto	10^2	h	6g
kilo	10^3	k	6j
mega	10^6	M	6#l
giga	10^9	G	6#f
tera	10^{12}	T	6#s
peta	10^{15}	P	6#o
eksa	10^{18}	E	6#d

Examples:

$$1 \text{ l} = 1 \text{ dm}^3 = 0.001 \text{ m}^3$$

$$1 \text{ km}^3 < 1 \text{ cl} / 9 < 10 \text{ l} / 9$$

$$0^\circ\text{C} = 273.15\text{K}$$

$$1 = (\text{b} < \text{a} \text{fb} 0^\circ \text{d} 6 \# \text{j}$$

$$0^\circ\text{C} = 32^\circ\text{F}$$

$$1 = (\text{b} < \text{ba} (\text{e}$$

$$100^\circ\text{C} = 32^\circ\text{F}$$

$$1 \text{ ii} (\text{b} < \text{a}^\circ \text{a} (\text{e}$$

$$\frac{3}{4}^\circ\text{C} = 0.75^\circ\text{C}$$

$$1 \text{ b.} = (\text{b} < 10 \text{fd} (\text{b}$$

$$2 \frac{2}{5}^\circ\text{C}$$

$$1 \text{ a}^\circ \text{a} > = (\text{b}$$

The second designation
°C is used in the records to avoid ambiguity.

$$1 \text{ Å} = 10^{-10} \text{ m}$$

$$1 \text{ Z} < 1 \text{ i} / , 0 (6 \text{ l}$$

$$1 \text{ w.} = 1 \text{ NM/h} = 1852 \text{ m/h}$$

$$1 \text{ v} < 1 \text{ m} \# 17 \text{ g} < 1 \text{ gda} 6 \text{ l} 7 \text{ g}$$

$$1 \text{ dB} = 10 \text{ B}$$

$$1 \text{ c} \# \text{a} < 1 \text{ i} 6 \# \text{a}$$

$$1 \text{ ha} = 10,000 \text{ m}^2$$

$$1 \text{ g}^\circ < 1 \text{ i} - \text{iii} 6 \text{ l} / :$$

Rules governing the use of the symbol

1. Verbal symbols preceded by the key symbol 6Z are taken from the black print without any changes.

Full name	Verbal symbol	Braille transcription	Additional abbreviations frequently used symbols
radian hour	rad	6OE`c	"OE
hectare	h	6g	
meter	ha	6g`	
candela	m	6l	
ampere	cd	6bc	
	A	6#`	

2. Within the symbol, each Greek letter or capital Latin letter refers only to the letter preceding it. When the word symbol begins with a lowercase Latin letter, the alphabet symbol is omitted, i.e., in such a case, no alphabet symbol is written.

5 kg 2 µm
 "d6jf "a6;l&l
 2Ah 3kWh
 "a6#`g g"b6j#vg

The rules given in rule 2 for word symbols differ from the rules for using alphabet characters in algebraic expressions (Guide section – *Alphabet characters*).

3. The word symbol together with the key character considered as a whole is of type A-B' ⁶⁴. This means that there may or may not be a blank space before it, but there must be a space after it.

entry without a space notation with a space

5 min "d6lhm "d□6lhm
 10 nF "`i6m#e                  "`i□6m#e

The notation without a space is more commonly used

Very often, physical units are composed of several verbal symbols. In such cases, instead of writing the unit symbol before each symbol, it is sufficient to write the double unit symbol 66Z before the entire unit. No spaces are required within a unit written in this way.

J = N m
 6#i□<6#m-6l□<66#M-l

$$J = \text{kg} \frac{\text{m}}{\text{s}^2} \cdot \text{m} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

 6#i□<6jf-6l7r/:□-6l□<
 <66jf-l/:7r/:

⁶⁴ Chapters of the Guide – *Sequence of characters, Mutual position of characters*.

4. The name sign is not used with currency names and abbreviations. The name of the currency or its abbreviation should be separated from the number by a space.

5 zlotys

5 zł

5 PLN

"d□y1nsxbg

"d□y1

"d□#O#K#M

3 PLN 50 gr

3.50 PLN

"b□y1□"di□fœ

"b0di□y1

5. The sign is also not preceded by units written in full. four seconds 3 kilograms

bysdœEx□rdjtmcx "b□jhknfœ`lx

1 hour 30 minutes

2 volts

"`□fncyhm`□"bi□lhmts "a□vnksx

Main units – examples

work

J = Ws

6#i□<6#v-6r or 6#i□<66#v-r

force

N = kg $\frac{m}{s^2}$ = $\frac{kgm}{s^2}$

The double unit symbol 66 can be used in expressions involving operations on units.

6#M□<6jf□-6l7r/:□<6jfl7r/:

or

6#M□<66jf-l7r/:□<6jfl7r/:

temperature units

The basic unit of temperature in the SI system is *the kelvin* [K] – 6#j However, the traditional unit, *the degree Celsius* [°C], is very often used.

°C z(#B or z=(#B degrees Celsius

20°C

36.6°C

$2\frac{2}{5}$ °C

"ai(#B

"be0e(#B

"a"a>=(#b

Units in formulas and expressions physical

Example

Calculate the capacitance of a flat capacitor whose plates are squares with sides of length $a = 10$ cm, if the distance between them is $d = 0.2$ mm.

Data:

$$a = 10 \text{ cm} = 0.1 \text{ m}$$

$$d = 0.2 \text{ mm} = 2 \cdot 10^{-4} \text{ m}$$

$$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{F}}{\text{m}}$$

The Braille transcription of the written data and the continuation of the solution will be as follows:

```
#c`md9
&`□<"`i6bl□<"i0`6l
&c□<"i0a6ll□<"a-"`i/, .6l
;d0(□<"g0gd-"`i/,0:6#e7l
```

$$C = \frac{\epsilon_0 S}{d} = \frac{8.85 \cdot 10^{-12} \frac{\text{F}}{\text{m}} \cdot 10^{-2} \text{m}^2}{2 \cdot 10^{-4} \text{m}} \approx 0.44 \cdot 10^{-9} \text{F} = 0.44 \text{nF}$$

```
#b□<;d0(#R7&c□<:"g0gd-"`i/,0:!!
!6#e7l-"`i/, :6l/:□7□"a-"`i/, .!
!6l;□>>"i0cc-"`i/,)6#E□<"i0cc!
!6m#E
```

The three consecutive lines in the above Braille notation have been broken at a place that is neither an operation sign nor a blank sign. Therefore, we place the sign ! at the end of the line and repeat this sign at the beginning of the next line.⁶⁵

In Braille notation, operations on numbers can be separated from operations on units. This notation seems to be more readable for blind people.

$$C = \frac{\epsilon_0 S}{d} = \frac{8.85 \cdot 10^{-12} \cdot 10^{-2} \frac{\text{F}}{\text{m}^2}}{2 \cdot 10^{-4} \text{m}} \approx 0.44 \cdot 10^{-9} \text{F} = 0.44 \text{nF}$$

```
#b□<;d0(#R7&c□<:"g0gd-"`i/,0:-
-"`i/, :□7□"a-"`i/, .;-66:#e7l□-
-l/:□7□l;□>>"i0cc-"`i/,)6#E□<
<"i0cc6m#E
```

⁶⁵ Chapter of the Guide – Transferring parts of mathematical expressions to the next line.

Braille chemical notation refers to mathematical notation. However, chemical symbols and formulas, as well as reaction notation, essentially express completely different content from mathematical expressions. Two-letter symbols for elements constitute a whole and certainly cannot be treated as mathematical expressions. The notation of chemical reactions is a representation of processes that are difficult to compare with mathematical transformations. Therefore, chemical notation can only be said to bear a certain resemblance to mathematical notation. The key symbols used in chemical notation and the applicable rules of notation are precisely specified and defined in the following chapters of the Guide.

BASIC SYMBOLS AND SIGNS USED IN CHEMISTRY

In the notation presented, the symbol Z is used to indicate the continuation of the notation without a space, and the space symbol (⠠) is used.

symbols for elements

The symbol of an element is preceded by *the element's key symbol*:

#Z	- element key symbol ⁶⁶			
H	Cl	Fe	S	Mn
#g	#Bk	#ed	#r	#Lm

formulas of molecules

In molecular formulas, the symbol of each element is preceded by *the element's key symbol*.

HCl	CaO	MgS	CuO
#g#Bk	#B`#N	#If#R	#Bt#N

If all the symbols of the elements in a chemical compound molecule are single letters, the molecular formula can be written in abbreviated form, omitting the symbols of the key elements. The molecular formula should be

but preceded by a key character ⁶⁷.

^Z - keyword for molecules with single-letter symbols for constituent elements

	abbreviated notation	full notation
KOH	^jng	#j#n#g
HCOOH	^gbnng	#g#b#n#n#g
HCN	^gbm	#g#b#m

⁶⁶ In mathematical and literary notation, this is *the key symbol for the capital letter of the Latin alphabet*.

⁶⁷ Based on the findings of the International Conference on Braille Notation in Moscow in 1975: *A System of Braille Notation of Mathematics, Physics, Astronomy and Chemistry*, Moscow 1975 [4].

nd stoichiometric indices

Stoichiometric indices in a molecular formula are written directly after the symbols of the elements in lowercase (fifth series characters).

CaSO₄Fe₂O₃MnO₂

#b`#r#n.

#ed:#N9

#lm#n:

If the molecular formula contains only elements with single-letter symbols, abbreviated notation may be used:

abbreviated notation

full notation

H₂O

^g:N

#G:#n

H₂SO₄

^g:Rn.

#g:#R#n.

Round brackets

We use parentheses in molecular formulas as in mathematical notation.⁶⁸

1Z - open round bracket

Z? - closed round bracket

Ca(OH)₂Al₂(SO₄)₃

#b`1#n#g?:

#`k:1#r#n.?9

's coefficients

Stoichiometric coefficients in chemical reaction equations are written with *the number sign*⁶⁹ without a space between the coefficient and the formula or symbol.

"Z - numerical sign

2H₂3H₂O2Al₂O₃

"a#g:

"b#g:#n

"a#@k:#N9

You can use the ^ symbol before the formula.

3H₂O2P₄O₁₀

"b^g:n

"a^o.n0(

⁶⁸ Guide Chapter – *Parentheses*.

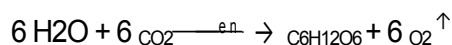
⁶⁹ Guide Chapter – *Numbers*.

Arrow symbols in chemical notation

←	□09z	- reaction proceeding to the left
→	□90z	- reaction proceeding to the right
↔	□090z	- reversible reaction
↑	^9`z	- gas evaporation
↓	^9-z	- sediment precipitation
$\xrightarrow{w \text{ yr.}}$	□;90&vxOE-	- right arrow written under the expression
$\xleftarrow{w \text{ yr.}}$	□;09&vxOE-	- left arrow written under the expression
$\xleftrightarrow{w \text{ yr.}}$	□;090&vxOE-	- right and left arrows written under the expression

-catalyzed reaction

When writing down a reaction involving a catalyst or requiring the specification of reaction conditions, a "long arrow written under the expression" is used. The expression is preceded by the & sign and ends with a - sign. After writing the expression, a blank character is required.



"e^g:n□*"e^bn:□;90&dm-□^b*g0:n*&
*"e#n:□^9`

When transferring part of the notation to the next line in the above example, & was used in accordance with the rules for transferring mathematical expressions.

The issue of carrying part of an expression over to the next line is discussed in detail in the Guide section – *Carrying part of a mathematical expression over to the next line*.

Indices 71

In chemical notation (electron configuration, valence, ions), left- and right-hand indicators are used, both upper and lower. These indicators have their own key characters in Braille.

Key characters for right-hand indicators

z{z - upper right-hand index
z0z - lower right-hand index Key

characters for left-hand indices

abbreviated notation	normal notation	
$\square\{Z$	" $\{Z$	- upper left-hand index
$\square OZ$	" OZ	- lower left-hand index

In general, abbreviated notation is sufficient. Standard notation for left-hand indices should be used in cases where ambiguity must be avoided.

⁷⁰ The arrow symbol is also used in physical formulas; see the section of the Guide entitled *Arrow symbols in physical notation*.

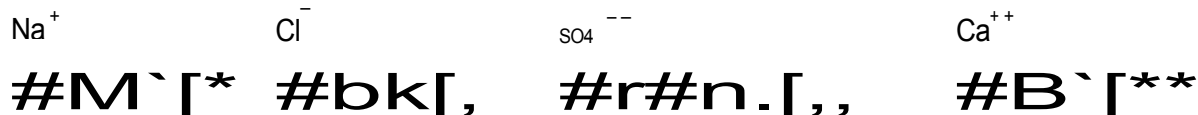
⁷¹ Guide section – *Powers and exponents*.

Ions

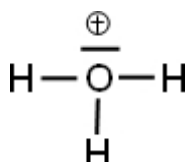
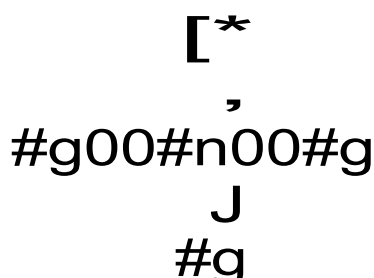
Ion charges are given after the right-hand superscript.

- $\{-$ - negative ion
 $\{^*$ - positive ion (abbreviated notation)
 $\{!^*$ - positive ion (full notation)

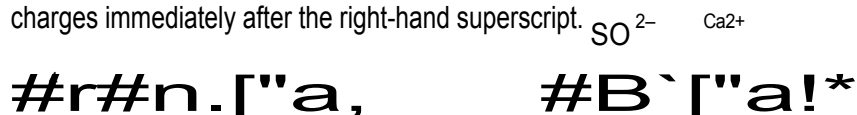
The full notation of a positive ion should be used in designations where there could be ambiguity in the reading.



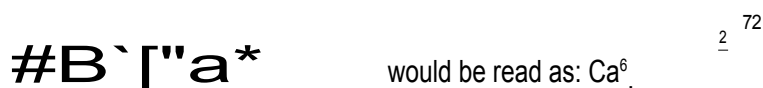
Hydrogen ion



Two-, three-, and more positive ions, as well as two-, three-, and more negative ions, can also be written by specifying the number of charges immediately after the right-hand superscript.



The full notation of the Ca^{2+} ion was used. If the abbreviated notation was used by mistake, the formula



Hydrates

In hydrated molecules, a *multiplication sign* is inserted between the formulas of the anhydrous molecules and water.

dot without space

dot with space



or

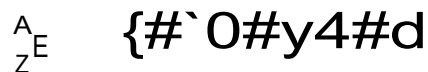


⁷² Guide Chapter – Common fractions.

⁷³ Chapter of the Guide – Operation symbols.

e structure of an atom

In the periodic table, each element is described by its atomic number Z (number of protons) and mass number A , placed before the element symbol as left-hand indices: A as the upper index and Z as the lower index ⁷⁴.



Indicators can be written in full or abbreviated form. In full form, numbers are written using *numeric characters*, while in abbreviated form, numeric characters are omitted and numbers are written in lower level (characters from the fifth series of the Braille alphabet).



The end of projector symbol must be placed before the root symbol. 4

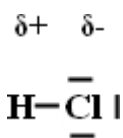
Electronegativity of elements

The difference in electronegativity between the atoms being combined should be written in accordance with the Greek delta symbol, giving the formula of the compound in question in the right-hand subscript.

$$\Delta E_{\text{NaCl}} = 3.0 - 0.9$$



Partial positive and negative charges should be denoted by the lowercase Greek letter delta with a + or – sign ($\delta+$ or $\delta-$). A separator sign ! should be placed between the letter and the plus or minus sign.



⁷⁴ Chapter of the Guide – *Left-hand indicators*.

Arrangement of valence electrons in an atom

0 - on the left side of the symbol

0 - to the right of the symbol

— - above the symbol

` - below the symbol

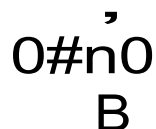
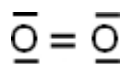
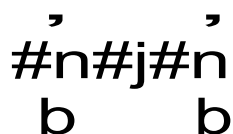
**non-bonding electron pairs**

⌘ - on the left side

⌘ - on the right

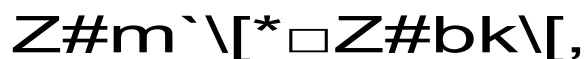
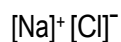
⌘ - from above

b - from below

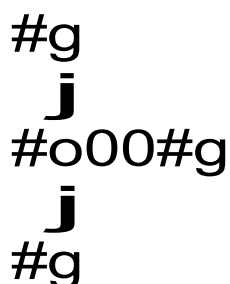
**Arrangement of electron pairs binding**

chemical bonds**Ionic bond**

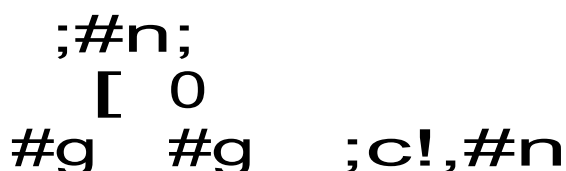
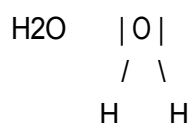
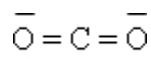
Each ion forming a bond is written in square brackets with the ion symbol and its value outside the brackets.



Covalent bond – notation analogous to the arrangement of bonding electron pairs.



In the case of a **polarized covalent bond**, the symbol of the more electronegative element preceded by the δ -sign is given according to the molecular formula.



Coordinative bond – All covalent bonds are written in the molecular formula. Then, after a blank space, the symbols of the elements forming the coordinative bond (donor-acceptor) are given. An arrow pointing towards the acceptor is placed between the symbols of the elements.

#g00#g



;#b#j#n #b09#n
b

electron configuration

The number of protons characteristic for a given element is written as a left-hand subscript before the symbol of the element. The number of electrons in each orbital is given as a right-hand superscript after the letter denoting the orbital. After the symbol of the element, a separator & should be inserted before the colon. If there is no separator, the notation can be read as ${}_{11}\text{Na}_3$.

${}_{11}\text{Na}$: 1s² 2s²p⁶ 3s¹

O"``#M`&9□"``&r{:□"a&r{:&o{*& "b&r{O

${}_{19}\text{K}$: 1 s² 2 s²p⁶ 3 s²p⁶ 4 s¹

O"``h#J&9□"``&r{:□"a&r{:&o{!
!*□"b&r{:&o{*□"c&r{O

If a mathematical expression is interrupted at a point where there is no empty cell, then the ! symbol is used as a carry symbol. This symbol must be repeated at the beginning of the next line.

n notation of an electron

In electron notation, a dash or minus sign can be used.

e &d9 3e "b&d9 notation with a dash "mark"

e- &d, 3e- "b&d, entry with a minus sign

Values

- In the notation of value, Roman numerals are written according to mathematical notation.⁷⁶
- The valence is given after the symbol of the element as a right-hand superscript.
- In the molecular formula, the symbols of successive elements are preceded by the projection termination sign 4.

Al^{III}

Ca^{II}O^{II}

#@k{#hhh #b`{#hh4#N{#hh

⁷⁵ Chapter of the Guide – So-called "marks."

⁷⁶ Chapter of the Guide – Roman numerals.

e oxidation states of elements

Roman numerals are used to denote the oxidation states of elements, placing them after the element symbol in parentheses. Negative oxidation states are indicated by adding a minus sign before the Roman numeral. If a stoichiometric coefficient appears in the molecular formula, the oxidation state is written after the coefficient.

Roman numeral notation

IV -II

S O₂

#r1#HU?#n:1,#HH?

For free elements, their oxidation state should be written as an Arabic numeral zero.

Full notation

⁰N₂ #M:1"i?

Abbreviated notation

⁰N₂ #M:1(?)

Full transcript

⁰O₂ #n:1"i?

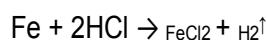
Abbreviated record

⁰O₂ #n:1(?)

CHEMICAL REACTIONS

Formation of salts

Molecular formula

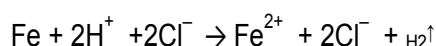


#ed□*"a#g#bk□90#ed#bk:□*#g:^9`

Ionic notation

The charge of an ion is written after the right-hand superscript.

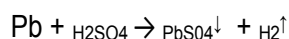
For ions with 2, 3, or more positive charges, the asterisk (*) must be preceded by a separator !.



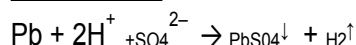
#ed□*"a#g[*□*"a#bk[,□90
90#ed["a!*□*"a#bk[,□*#g:^9`

Formation of an insoluble salt

Molecular formula

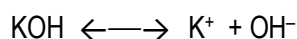


#Oa□*^g:rn.□90^OaRn.^9□*#g:^9`

Ionic notation

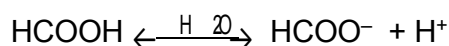
#Oa□*"a#g[*□*^rn.["a,□90
90^OaRn.^9-□*#g:^9`

reversible reaction



^JNG□090#j[*□*^NG[,

dissociation reaction



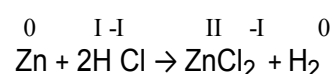
^gbnng□;090&^g:n-□^gbnn[,□*#g[*

In the reaction notation, a long arrow pointing right and left is used under the expression⁷⁷.

;090&^g:n- - a long arrow pointing right and left written under the expression.

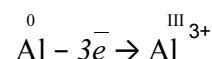
Oxidation and reduction reactions

Oxidation states in redox reactions are written exclusively in Roman numerals after the element symbol in parentheses.

Roman notation

The oxidation states in the following examples are given in abbreviated form.

#ym1(?□*"a#g1#H?#bk1,#h?□90
90#ym1#HH?#bk:1,#h?□*#g:1(?

Electron balance in oxidation and reduction reactions

#`k1(?□,"b&d9□90#@k["b!*1#hhh?

Reaction sequence diagram

Chemistry problems often use reaction sequences for students to decipher. The "+" sign indicates a compound added to the reaction, while the "-" sign indicates a compound eliminated from the reaction.



#`□1*#bk: ?□90#a□1,#g#Bk?□90#b

⁷⁷ Chapter of the Guide – Arrow symbols in chemical notation.

Notation of units [chem.] ⁷⁸

In chemical notation, units are written in accordance with mathematical and physical notation. The unit is preceded by a symbol.

6z - unit symbol, name symbol.

g mol dm³ g/mol
6f **6lnk** **6cl/9** **6f7lnk**

12g 5.6 mol 3g/dm³
"`a6f **"d0e6lnk** **"b6f7cl/9**

Percentage concentration of a solution

The following formula is used to calculate the percentage concentration of a solution:

$$c_p = \frac{m_s}{m_r} \cdot 100\%$$

&b0o□<lOr7lOœ□-"`ii"i(

When calculating the percentage concentration of a solution formed by mixing solutions of different concentrations, the following formula is used:

$$c_{p_1} = \frac{m_{s_1}}{m_{r_1}} \cdot 100\%$$

&b0o00□<lOr007lOœ00□-"`ii"i(

or full notation

&b0o00□<:lOr00□7 □lOœ00;- "`ii"i(

The example uses symbols used in writing fractions⁷⁹:

:Z - beginning of the fraction (numerator)

;Z - end of the fraction (denominator)

□7□ - fraction line

and right-hand indicator sign⁸⁰:

z0z

⁷⁸ The notation of units is described in detail in the chapter of the Guide – *Units and operations on units and Verbal symbols, denominator sign.*

⁷⁹ Chapter of the Guide – *Fractions.*

⁸⁰ Chapter of the Guide – *Key symbols for exponents and right-hand indices.*

E GENERAL FORMULAS

In organic chemistry, general formulas are used for groups of compounds, e.g., hydrocarbons, alcohols.

C_nH_{2n+2}

#b0&m4#g0"a&m!*"a

The ! character is a placeholder.

$C_nH_{2n+1}OH$

#b0&m4#g0"a&m!*"4#n#g

SEMI-STRUCTURAL AND STRUCTURAL FORMULAS

Bonds between atoms

z00z horizontal single

z

j

vertical single

z

z#jz horizontal double

z

in

vertical double

z

z^k triple horizontal [zz

right-angled single

z0z left-handed single

z[[z right-handed double

z00z left-handed double

Structural formulas of inorganic compounds

HCl	H – Cl	#g00#bk
CO	C = O	#b#j#N
N ₂	N ≡ N	#M^k#M
Ca(OH) ₂	HO – Ca – OH	#g#n00#b`00#n#g
or	H – O – Ca – O – H	#g00#n00#b`00#n00#g
Al ₂ O ₃	O = Al – O – Al = O	#n#j#`k00#n00#@k#j#n
$\begin{array}{c} \text{O} = \text{S} = \text{O} \\ \\ \text{O} \end{array}$		#N#j#R#j#N in #N

Structural and semi-structural formulas of organic compounds .

Hydrocarbon formulas are written in total and semi-structural form.

For long hydrocarbon chains, the notation can be shortened by placing the CH₂ group in parentheses and writing the number of these groups in subscript after the parentheses. If it is an isomeric compound, the substituent should be placed in parentheses, without a bond sign next to the carbon atom to which it is attached. The proposed Braille notation allows for the notation of two substituents on the same carbon atom or the notation of a substituent with an additional substituent.

C₂H₅COOH propanoic acid

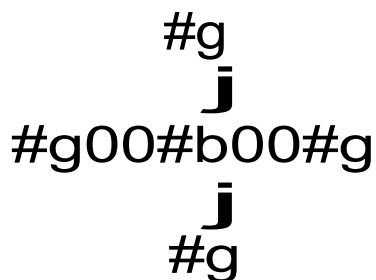
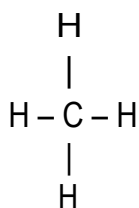
CH₃ – CH₂ – COOH

^bg900^bg:00^bnng or
#b#g900#b#g:00#b#n#n#g

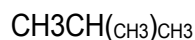
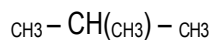
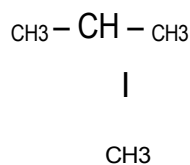
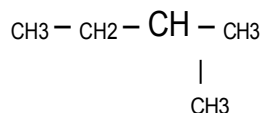
C₁₀H₂₂ n-decane

CH₃ – (CH₂)₈ – CH₃

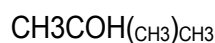
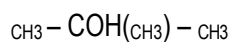
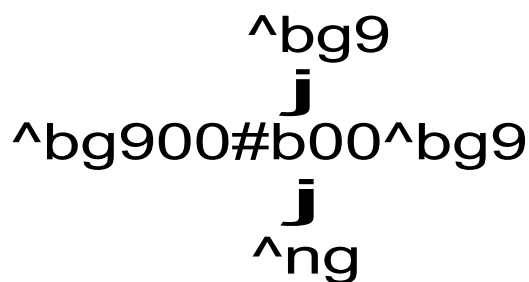
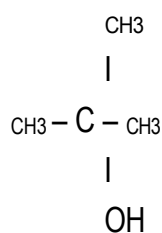
^bg9001^bg:?'00^bg9

CH₄ methane:

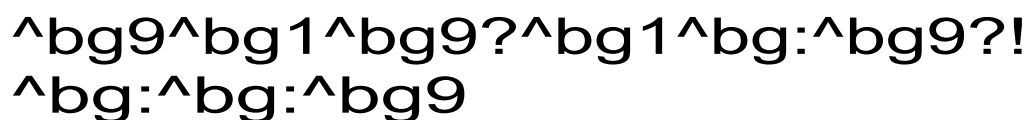
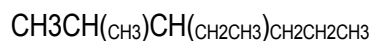
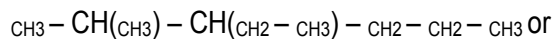
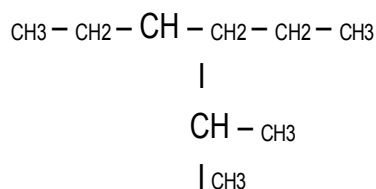
2-methylpropane (isobutane):

C₅H₁₂ 2-methylbutane:

2-methyl-propan-2-ol:



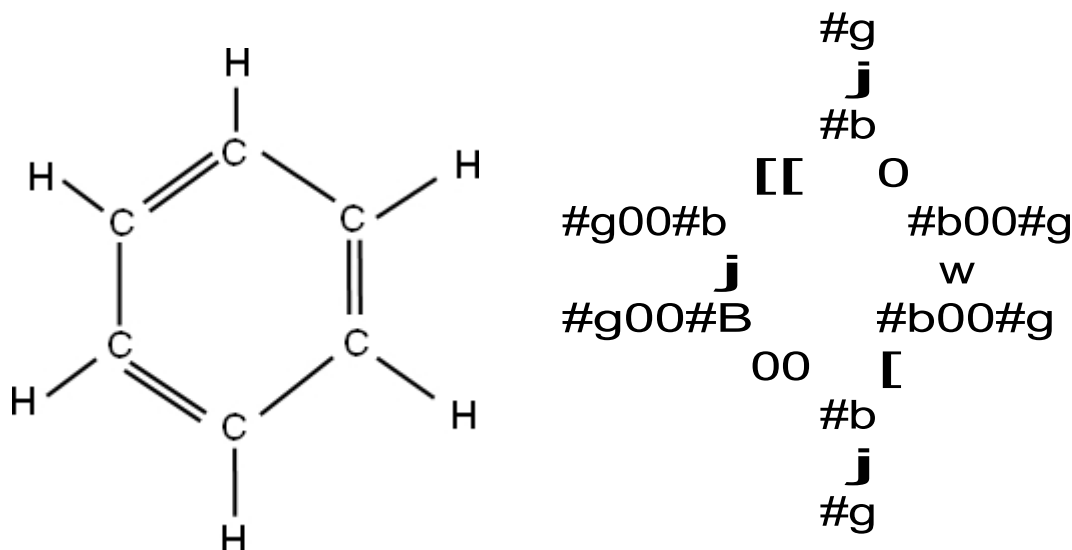
3-ethyl-2-methylhexane



e ring compounds

In the structural formulas of cyclic compounds, horizontal, vertical, and right- and left-handed bonds are used.

Benzene – structural formulas



Three levels of information are used in Braille notation for ring compounds:

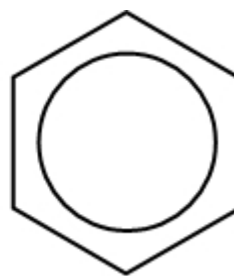
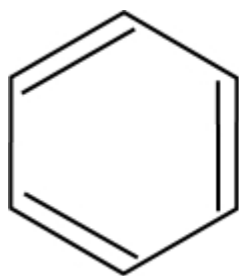
1. The ring compound and the number of atoms forming the ring.
2. The location and type of substituents.
3. The position of double bonds.

The following characters are used for this purpose:

%	ring structure compound (single ring or composed of several rings connected by bonds)
\$	multi-ring compound (the rings are connected by common carbon atoms)
#o	in combination with a number, indicates the number of atoms forming the ring
4	closing mark for a fragment of information
04	composite closing mark for the entire formula
%#`	aromatic compound
":	second level of
information "9	third level of
information	

Benzene – one of the forms of Kekulé's formula

simplified formula



%#o"e4
"9□#j":b.d*`04

%#`o"e04

Level 1: indicates the ring structure of the compound.

%#o"e means that it is a ring compound with a six-membered ring.

%#`o"e04 means that it is a six-membered aromatic compound.

Level 2: not used because there are no substituents.

Level 3: indicates the position of double bonds in the chain.

The entry begins with the information level number "9", followed by a space, then a double bond symbol, and then, without a space, the numbers of the carbon atoms between which the double bond is located (the first digit is lowered, the second is not lowered, and the following digits are lowered according to the same rule)

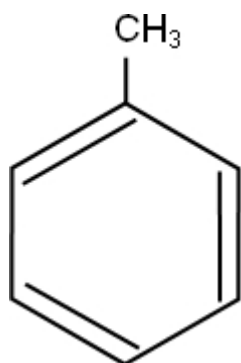
e.g. #j":b.d*` means that the double bonds are between carbon atoms 2 and 3, 4 and 5, and 6 and 1, respectively.

Levels 1 and 2 of information end with the closing

symbol 4. The entire formula ends with the compound

closing symbol 04.

Toluene



%#o"e4
":□"01^bg9?4
"9□#j"-a9c>e04

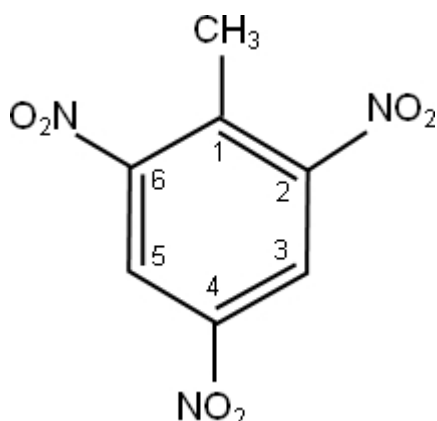
When writing down the formulas of benzene derivatives, it is possible to use a simplified formula specifying the substituents and their locants (2 levels of information).

Level 2: provides information about the location and type of substituents.

The notation begins with the information level number ":", followed by a space, then the number of the carbon atom to which the substituent is attached, followed by the substituent formula in parentheses without a space.

For example, "01^bg9?4" means that a methyl group is attached to the first carbon atom. Subsequent substituent formulas are written analogously and without spaces.

2, 4, 6 – trinitrotoluene (TNT, trotyl)



%#o"e4 ":\ "01^bg9?":1^mn:?.1^mn:?!
!"*1^mn:?4
"9\#j"0a9c>e04

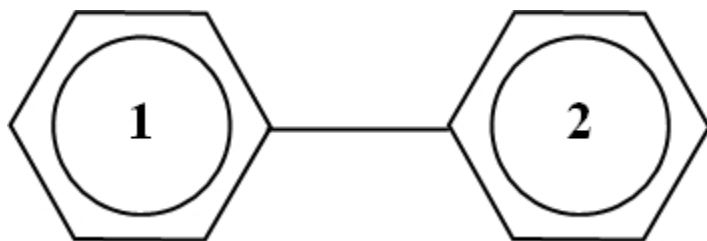
If there are more identical substituents, the notation can be shortened by giving the carbon atom number with the substituent formula after the number in subscript. The substituent formula enclosed in parentheses ends with a subscript number indicating the number of substituents.

%#o"e4
":\ "01^bg9?":". "*1^mn:?94
"9\#j"0a9c>e04 or

simplified notation

%#`o"e4
":\ "01^bg9?":". "*1^mn:?904

Diphenyl - the two rings are connected by a bond and do not share a side, so the Braille formula begins with the % sign.



%#`o"0e00#`o":e04

Multi-ring compounds

In Braille, each ring in a polycyclic compound is numbered. In each ring, the carbon atoms are numbered clockwise starting from the top carbon atom.

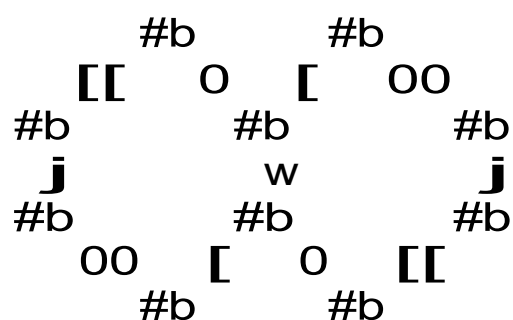
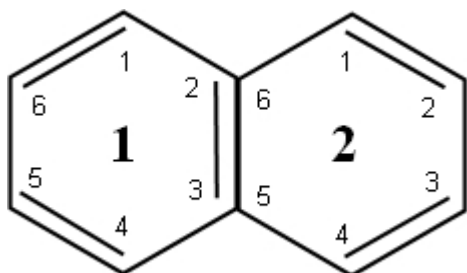
The notation begins with the sign **#0**, followed by the number of the ring being described.

in lowercase notation, without a space. Then, between the **U** characters, the number of the adjacent ring is given in lowercase. Next, the numbers of carbon atom pairs are given, listed clockwise, in the same notation as for bonds (the first resulting from the numbering in the described ring – in subscript, the second resulting from the numbering in the adjacent ring – in superscript).

For greater clarity, the description of subsequent rings begins on a new line.

If a substituent is attached to the ring, the ring number is given, followed by the number of the carbon atom to which the substituent is attached.

Naphthalene – a bicyclic compound – full notation.



```

$a#o":e&
  #o"0eU:":e9d4
"9  #o"0e#j":b.d*`&
    #o":e#j"0a9c>e04
  
```

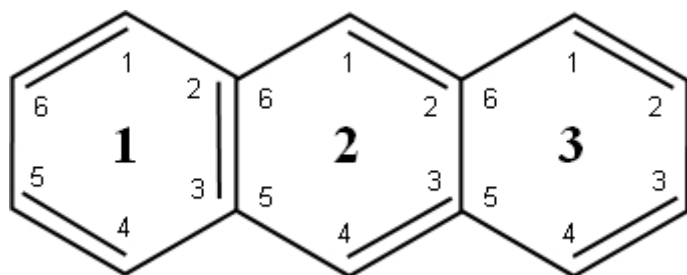
Naphthalene – abbreviated notation.

To define a shared side, only the number of the carbon atom starting the shared side is given. Then, the first shared pair of carbon atoms is given in the same notation as before (top, bottom). When describing rings, the number of ring members is omitted.

```

$a#o":e&
  #o"0U:":e4
"9  #o"0#j":b.d*`&
    #o":#j"0a9c>e04
  
```

Anthracene – a tricyclic compound – full notation.



\$"b#o"9e&

#o"0U:":e9d& #o":U0">b*aU9":e9d&

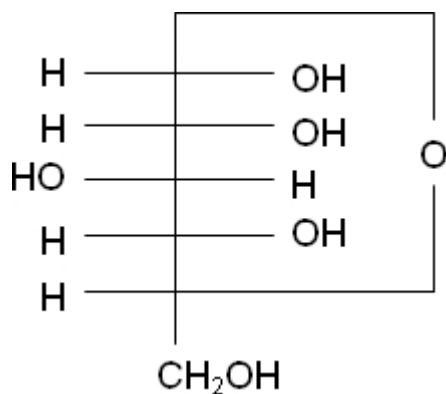
#o"9U:">b*a4

"9 #o"0#j":b.d*` &

#o":#j"0a9c>e&

#o"9#j"0a9c04

Glucose: α -D-glucopyranose – ring form – Fischer notation.



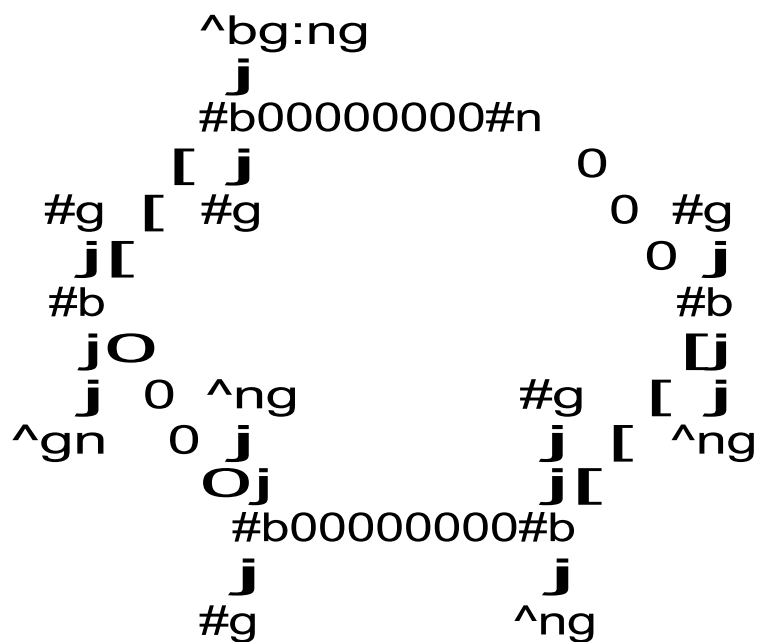
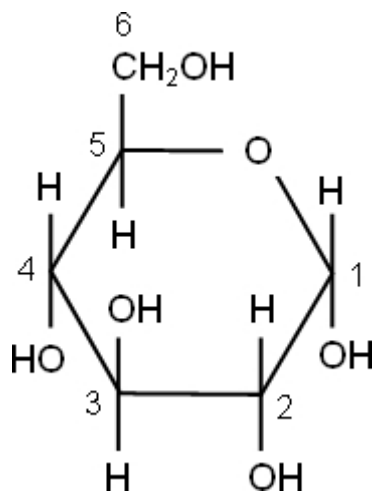
A row of dots is used to close the ring of a monosaccharide molecule in Braille.

```

0000000000000000
  J                                     J
#G00#B00^NG                               J
  J                                     J
#G00#B00^NG                               J
  J                                     #N
^GN00#B00#G                               J
                                     J
#G00#B00^NG                               J
                                     J
#G00#B0000000000000000 J
  ^BG:NG

```

Glucose: α -D-glucopyranose – ring form – Haworth notation.



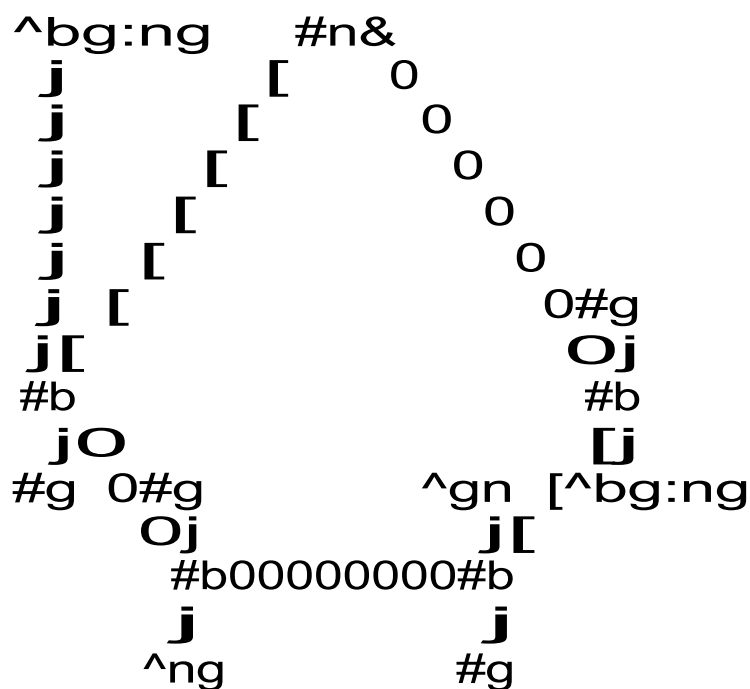
%#o"e4

```

": "010^ng?1{#g?":10^ng?1[#g?!
!"910#g?1{^ng?".10^ng?1{#g?!
!">10#g?1{"*^bg:ng?00#n04

```

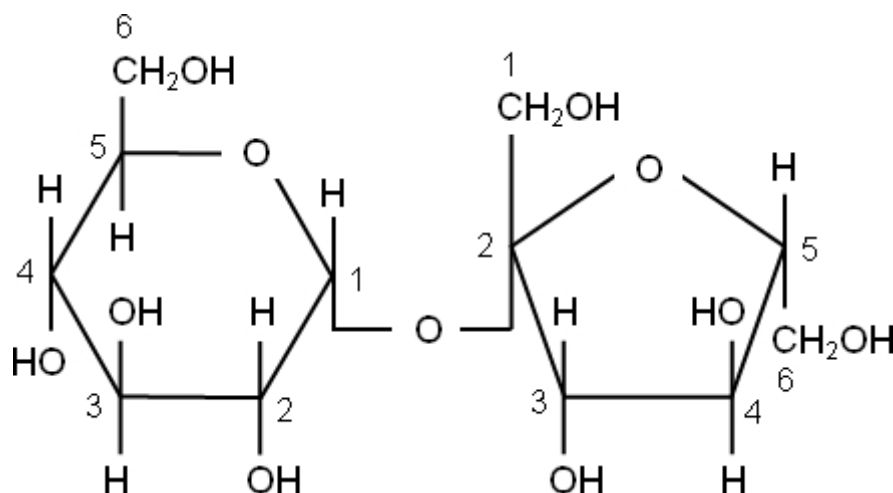
Fructose - α -D-fructofuranose - carbon atoms are numbered counterclockwise (to the left). In Braille, this is indicated by the sign 09, understood as an auxiliary arrow.



%#o"d4

": 09":10#g?1["0^bg:ng?!
!"910^ng?1[#g?".10#g?1[^ng?!
!"d10"*^bg:ng?1[#g?00#n04

Sucrose – in the fructose ring, carbon atoms are numbered counterclockwise (to the left). This is indicated by the symbol 09, understood as an auxiliary arrow.



%#o"0e00#n00#o":d4

"": #o"0e"010#n#o":d?1{#g?!
 !":10^ng?1[#g?"910#g?!
 !1[^ng?".10^ng?1[#g?!
 !">10#g?1["*^bg:ng?00#n&
 #o":d09":10#n#o"0e?1["0!
 !^bg:ng?"910^ng?1[#g?!
 !".10#g?1[^ng?">10!
 !"*^bg:ng?1[#g?00#n0

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⁸¹ Entries marked additionally with [chem.] refer to chemical records, and those marked with [phys.] refer to physical records.

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